

**DEVELOPING DIFFERENT  
FORMULATIONS OF SSCWLP  
(SINGLE STAGE CAPACITATED WAREHOUSE  
LOCATION PROBLEM)  
&  
EMPIRICALLY ESTABLISHING RELATIVE  
STRENGTHS OF MANY OF ITS RELAXATIONS**

**A Thesis Submitted  
in partial fulfillment of the requirements  
for the degree of**

**MASTER OF TECHNOLOGY**

**by  
VISHAL BERRY**

**to the**

**DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
JANUARY 2003**

30 MAY 2003  
पुस्तकालय केन्द्रीय केंद्र, काजब  
भारतीय प्रौद्योगिकी संस्थान कानपुर

अवधि क्र० A-143478

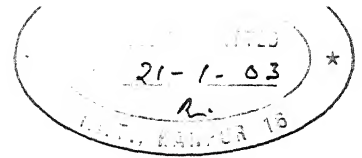
TH

IME/2003/M

B459d



A143478



## CERTIFICATE

It is certified that the work contained in the thesis entitled **DEVELOPING DIFFERENT FORMULATIONS OF SSCWLP (SINGLE STAGE CAPACITATED WAREHOUSE LOCATION PROBLEM) & EMPIRICALLY ESTABLISHING RELATIVE STRENGTHS OF MANY OF ITS RELAXATIONS**, by VISHAL BERRY, has been carried out under my supervision and that the work has not been submitted elsewhere for a degree.

**Dr.R.R.K. Sharma**

~~Associate~~ Professor

Department Of Industrial and Management Engineering

Indian Institute of Technology, Kanpur-208016

JANUARY 2003

January 2003

## **ACKNOWLEDGEMENTS**

Words wane in communicating my heartfelt gratitude to my thesis supervisor Dr. R.R.K. Sharma, who has devoted innumerable hours to the creation of the present work, and participated with full cooperation in the process of its refinement. Their critical evaluation of the manuscript spotlighted more than once the clumsy structure and logical gaps. The text is filled with echoes of the countless sessions of brain-storming with him. Through his superb guidance and esteemed supervision the present work has taken its shape.

My gratitude is also due to all the teachers of I.M.E. department for their superb teaching and kind help during the completion of my M.Tech work at IIT Kanpur.

My warmest thanks go to all my friends especially Mr. Sameer Sharma and Mr. Arun Pratap Singh whose company made my stay at Kanpur really enjoyable and working with them gave me the ability to think in a wider perspective.

Finally I take this opportunity to thank my parents and my other family members for keeping the faith on me through all these years of struggle and hard work. It wouldn't have been possible without their support and encouragement.



## ABSTRACT

The Single Stage Capacitated Warehouse Location Problem (SSCWLP) has been formulated in two different Styles as given by Geoffrion and Graves (1974) and Sharma (1991). We have used the two styles and combined them with the ideas of strong and weak relaxations ( as picked up from the formulations of capacitated plant location problems) to develop variety of new formulations of SSCWLP. Later we also added Big-M constraints that link the real variables (distribution variables) with the 0-1 integer variables (location variables).

We empirically established the strengths of Linear Programming relaxation of various formulations developed above. Among significant findings we found that Big-M formulations significantly boost the bounds (over and above the bounds given by WRS and SRS) for the case when  $\sum \text{cap}_j = 2.0$  and the constraint  $\text{cap}_j y_j \geq \sum_{ik} x_{ijk}$ . However it is not true when  $\sum \text{cap}_j = 6.0$  or  $\text{cap}_j \geq \sum_{ik} x_{ijk}$

Another Significant finding is that for solving linear programming relaxations of all kinds, the style of Geoffrion and Graves (1974) took significantly more computational iterations than the style of Sharma (1991).

We also found that the constraint  $\text{cap}_j y_j \geq \sum_{ik} x_{ijk}$  improved the bounds substantially in all possible formulations.

## CONTENTS

- *Acknowledgement*
- *Abstract*
- *List of tables*

### **Page No.**

1. INTRODUCTION	1
2. LITERATURE REVIEW	
2.1 Early Heuristics	5
2.2 Genetic Algorithm	6
2.3 Bender's Decomposition	8
2.4 Lagrangean Relaxation	17
2.5 Simulation Based Approach	21
2.6 Discussion	23
3. NEW FORMULATIONS FOR SSCWLP	
3.1 Problem Formulation ( Style 1 and Variation 1)	
3.1.1 Constant Definition	24
3.1.2 Variable Definition	25
3.1.3 Mathematical Formulation	25
3.2 Problem Formulation ( Style 1 and Variation 2)	31
3.3 Problem Formulation ( Style 2 and Variation 1)	
3.3.1 Constant Definition	33
3.3.2 Variable Definition	34
3.3.3 Mathematical Formulation	34
3.4 Problem Formulation ( Style 2 and Variation 2)	39
3.5 Summary	42
4. STRUCTURE OF EMPERICAL INVESTIGATION CARRIED	
4.1 Introduction	43
4.2 Computational Details	44
4.3 Results	46
5. ANALYSIS OF DATA AND RESULTS	
5.1 Introduction	47
5.2 Statistical Significance of Big M Formulations	48
5.3 Statistically Insignificant Difference of Bounds for .1B, .3B, .5B	52

5.4 Better Bounds by Variation 1 than Variation 2	54
5.5 Comparison of Number of Iterations for both the Styles	58
6. NEW FORMULATION	62
7. DIRECTION FOR FUTURE RESEARCH	64
REFERENCES	65
APPENDICES:	
APPENDIX I: Computational Results for all Formulations	
APPENDIX II: Computational Results for New Formulation	
APPENDIX III: Lingo Formulations for all Styles	

## LIST OF TABLES

TABLE NO.	TITLE	PAGE NO.
Table 4.1	Problems for Problem Size 5X5X5	44
Table 4.2	Problems for Problem Size 10X10X10	44
Table 4.3	Problems for Problem Size 20X20X20	45
Table 5.1	T-Test Between .1B and .2B $\Sigma cap_i = 2.0$	48
Table 5.2	T-Critical values data size 90	48
Table 5.3	% Improvement of Big-M formulation over SRS formulation of SSCWLP in Variation 1	49
Table 5.4	T-test between .1B and .2B $\Sigma cap_i = 6.0$	51
Table 5.5	F-Test between .1B, .3B and .5B	53
Table 5.6	F-critical values data size 90	54
Table 5.7	T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 5X5X5	55
Table 5.8	T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 10X10X10	55
Table 5.9	T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 20X20X20	56
Table 5.10	T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 5X5X5	58
Table 5.11	T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 10X10X10	59
Table 5.12	T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 20X20X20	60
Table 6.1	T-test for the New Formulation	63
Table 6.2	T-critical data size 50	63

## LIST OF FIGURES

FIGURE NO.	TITLE	PAGE NO.
FIGURE 1	Sketch of Comprehensive Distribution Planning System	3

## Chapter 1

### INTRODUCTION

A commonly occurring problem in distribution system design is the optimal location of intermediate distribution facilities or the warehouses between plants and customers. A capacitated single period version of this problem is formulated as a mixed integer linear programming problem in differing styles in literature.

The formal definition of the problem considered in this work and referred as a Single Stage Capacitated Warehouse location problem can be given as below. The problem SSCWLP arises when the distances between plants and markets are large and it becomes necessary to route the supplies through warehouses known as trans-shipment points. The set of potential trans-shipments locations are known and each point has an associated fixed cost with it. The problem is to choose a sufficient number of trans-shipment points such that the sum total of fixed location costs and transportation cost of shipping goods to market is (from plant to warehouse and from warehouse to market) minimized while satisfying the demands at each market points. It is assumed that the warehouses have finite capacity and which is known quantity. Also only a single commodity is considered for distribution.

Or in other general facility location is the problem of locating a number of facilities from a subset of  $m$  potential facility locations to completely satisfy at minimum cost all the demands of a set of  $n$  clients. Each client has an associated demand and the costs considered include transportation and fixed costs for opening facilities. For the *Uncapacitated Warehouse Location Problem (UWLP)* the facilities are assumed to have

unlimited capacity, and for the *Capacitated Warehouse Location Problem (CWLP)* there are constraints on the total demand that can be met from a facility.

Both the problems are well known to be NP-hard.

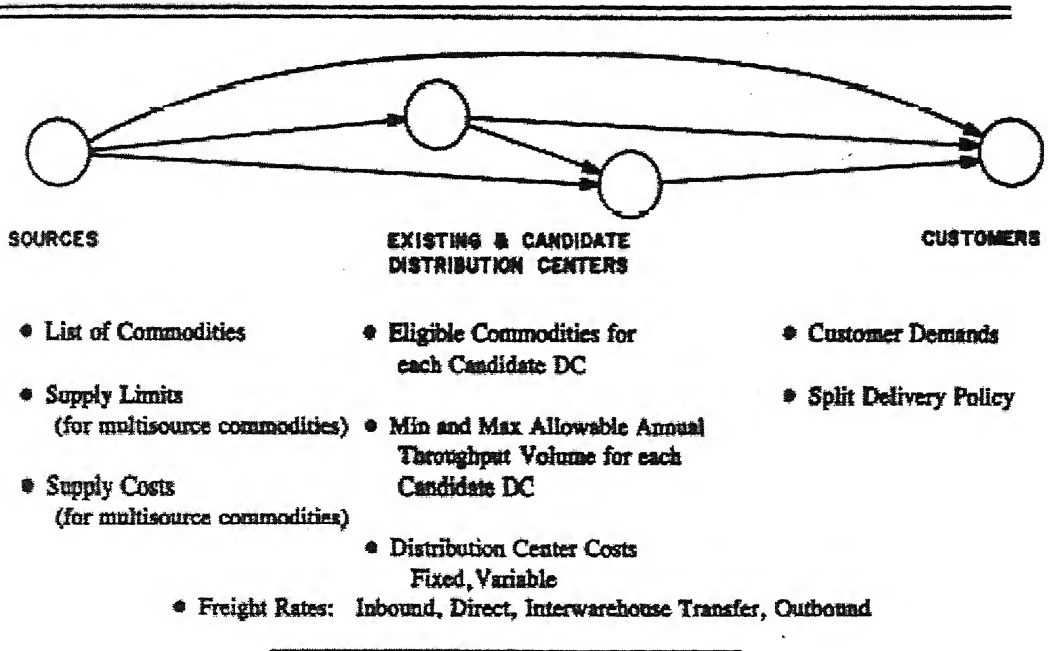
That is, if the number of candidate warehouses ( $m$ ) and customers ( $n$ ) are growing larger, the number of constraint equations for Integer Programming explodes and it is very hard not only to solve the problem, but even to represent it in the memory of a computer. Therefore, we need a practical method for finding an approximately optimal number and locations from a large number of possible locations.

So a multicommodity warehouse location problem may have a lot of aspects to consider such as supplying costs, supplying limits, list of commodities, list of eligible warehouse locations, minimum and maximum limits of throughput for the facilities, fixed cost of setting up the warehouse, customer demands, transportation or freight costs etc.

These considerations add up to the complexity of the model so a practical method is needed for real life large sized problems so that sufficiently closer bounds can be obtained with relatively less computational efforts.

To explain the requirement and function of the solver the diagrammatic sketch of a warehouse location problem is given as below.

**FIGURE 1. SKETCH OF A COMPREHENSIVE DISTRIBUTION PLANNING SYSTEM.**



### MAIN FUNCTION OF THE SOLVER

In this work the Single Stage Capacitated Warehouse Location Problem (SSCWLP) is formulated by two different styles. In style 1 the commodity is considered to be distributed from the supply locations to the markets in one stage. Style 1 is given in literature by Geoffrion and Graves (1974) [1]. In style 2 two stages are considered once the commodity goes from the plants to the warehouse and in the second stage from the warehouse to the markets. This style is given in literature by Sharma (1991) [2]. For each of the two given styles many relaxations are given and is empirically seen that a new relaxation (Big M method) gives a better bound than all the relaxations tried.



The organization of the thesis work is as follows:

In chapter two, we have given the relevant literature review. In chapter three we list down all the formulations for the SSCWLP for both the styles. In chapter four the computational experience and results for problems of various sizes are solved. In chapter five the analysis of the data in chapter four is done to establish which relaxation performs better and when and also the conclusion is given. In chapter six we have included a new formulation after getting good results from one of our formulations and the computational experience and analysis of the data for this formulation. Chapter seven gives the direction for future research on the topic of interest.

The formulations have been solved by using the optimization package LINGO and the objective values, and no. of iterations needed are noted for the various formulations. Approximately 2200 problems of various sizes were solved and each problem was randomly generated for this purpose.

## Chapter 2

### LITERATURE REVIEW

In this chapter, we give a brief literature review on SSCWLP (Single Stage Capacitated Warehouse Location Problem).

#### **2.1 Early Heuristics**

One of the earliest methods proposed for warehouse location/ facility location is the now well known heuristics proposed by *Kuhen & Hamburger (1962)* [13]. It has been widely studied and its catalytic effect can hardly be overrated. The heuristic consists of two parts. The main program is an ‘add routine’ whereby facilities are located one by one corresponding to the greatest cost reduction until no facility can be added without increasing the total cost. The underlying hypothesis is that the optimal solution for ‘(p+1)’ facilities can be determined from the optimal solution for ‘p’ facilities by adding an additional facility to the existing solution. In the modern technology, such a scheme would be called greedy algorithm because of its appetite for maximum improvement at each step. Upon termination of the main program, a so called bump and shift routine is entered. It first eliminates (bumps) any facility which is now uneconomical because of the proximity of another facility located subsequently. It also considers relocating (shifting) each facility from its actual location to other potential locations in its neighborhood. This was first applied to simple plant location problem; and possibly can be applied to SSCWLP.

## 2.2 Genetic Algorithm

Genetic Algorithms (GA's) are robust and adaptive methods that may be used to solve the problems related to search and optimization. GA's work with a population of individuals (usually 10-200), each representing a possible solution for the given problem. To each individual fitness value has assigned according to adaptation of this individual. The population evolves towards better solutions by means of randomized processes of selection, crossover, and mutation. The selection mechanism favors individuals of better fitness value to reproduce more often than the worse ones when forming a new population. The crossover allows mixing of parental information when it is passed to their descendants. The result of crossover is structured and randomized exchange of genetic material between solutions, with the possibility that "good" solutions can generate "better" ones. The mutation changes a bit in the binary string with some small probability  $pm$ . The role of mutation in GA's is restoring lost or unexplored genetic material into the population. It can be used to prevent the premature convergence of GA to suboptimal solutions. The initial population is usually randomly initialized.

*Kratica, Filipoviæ and Tosiæ (1998)* [6] have used a new hybrid approach for solving the Warehouse Location Problem by Simple Genetic Algorithm (SGA) and Add-Heuristic. The best individual in every generation of SGA is improved by Add-Heuristic, adding or removing one warehouse with maximal reduction of overall cost. The existence of warehouse with adding/removing may reduce overall cost and binary string is changed in the corresponding position. SGA itself produces good results with the reasonable running time, but hybrid approach improves performance of implementation. The local search nature of Add-Heuristic provides achieving better

regions of search for given problem. The GA implementation assisted with Add-Heuristic successfully prevents possibility of premature convergence, and restores lost or unexplored genetic material into the population. During the testing process, in both cases (base GA and hybrid GA) for every particular test problem at least one-half of optimal solutions are obtained. Both implementations are able to solve large-scale warehouse location problems, with good running time. According to WLP benchmark, the use of hybrid GA technique brings some improvements. In general, the average run-time produced by hybrid GA implementation is smaller than the one produced by the base GA implementation. The quality of solutions and average number of generations are also better for hybrid GA implementation.

For the given problem binary coding of possible warehouse locations is used. Every bit in the binary string denotes that particular warehouse is established if its value is 1, while 0 denotes it is not established. For the faster execution the item string is allocated in 32-bit words. The objective value function is computed depends on number of established warehouses  $e$  and threshold  $e0$ . If  $e > e0$  in initialization part, warehouses are indexed in non decreasing order of transportation costs for every customer. After that the objective value function speedily finds first established warehouse, with minimal transportation cost. If  $e \leq e0$  previous procedure is not appropriate, and ordinal array of established warehouses is formed. We find the established warehouse with minimal cost searching only the same ordinal array for every customer. The whole population is replaced in every generation except the best individual. The best individual in the current generation directly goes to the next generation, without selection, crossover and mutation. The initial population is randomly initialized, because the maximal diversity into the population is maintained in that case. The duplicate strings are discarded from the population, i.e. multiple occurrence of the same string is practically excluded. The

duplicate strings in every generation are discarded by setting their fitnesses to zero. This method does not remove duplicate item string physically, but eliminates its occurrence in the next generation. This technique maintains the diversity of genetic material. Discarding of duplicate strings decreases the appearance of dominating individuals in the population, and effectively decreases the possibility of premature convergence in local optimum. This is a very important factor for successful working GA, particularly in the cases of large size test problems.

### 2.3 Benders Decomposition

*Geoffrion and Graves (1974)* [1] one of the earliest researchers in the area of interest have formulated a multicommodity capacitated version of the problem as a mixed integer linear program. A solution procedure based on Bender's Decomposition is developed, implemented and successfully applied to a real problem for a major food firm. An essentially optimal solution was found and proven with a surprisingly small number of Bender's cuts. The major conclusions arising from the study is the remarkable effectiveness of Bender's Decomposition as a computational strategy for static multicommodity intermediate location problems. The numerical experience quoted also shows that only a few cuts are needed to find and verify the solution within one or two tenths of the global optimum.

The mathematical formulation of the problem uses the following notation

'i'	Index for commodities
'j'	Index for plants
'k'	Index for possible distribution centre or warehouse

'l' Index for customer demand zones

$S_{ij}$  Supply (production capacity) for commodity 'i' at plant 'j'

$D_{il}$  Demand for commodity 'i' in customer zone 'l'

$\overline{V_k}, \underline{V_k}$  Maximum and minimum allowed total annual throughput for warehouse at site 'k'

$f_k$  Fixed set up and operating cost for the warehouse at site 'k'

$v_k$  Variable unit cost of throughput for a warehouse at site 'k'

$C_{ijkl}$  Average unit cost of producing and shipping commodity 'i' from plant 'j' through warehouse 'k' to customer zone 'l'

$x_{ijkl}$  A variable denoting the amount of commodity 'i' shipped from plant 'j' through warehouse 'k' to customer zone 'l'

$y_{kl}$  A 0-1 variable that will be 1 if warehouse 'k' serves customer zone 'l' 0 otherwise

$z_k$  A 0-1 variable that will be 1 if a warehouse is acquired at 'k' 0 otherwise

$$\text{MINIMIZE} \sum_{ijkl} x_{ijkl} \cdot C_{ijkl} + \sum_k \left[ f_k z_k + v_k \sum_{il} D_{il} y_{kl} \right]$$

SUBJECT TO--

$$\sum_{k,l} x_{ijkl} \leq S_{ij} \forall ij \quad (1)$$

$$\sum_j x_{ijkl} = D_{il} y_{kl} \forall ikl \quad (2)$$

$$\sum_k y_{kl} = 1 \forall l \quad (3)$$

$$\underline{V}_{kZk} \leq \sum_{il} D_{il} y_{kl} \leq \overline{V}_{kZk} \forall k \quad (4)$$

Linear configuration constraints on y and/or z (5)

Constraint 1 being the supply constraint, constraint 2 stipulates both that the legitimate demand must be met (when  $y_{kl} = 1$ ) and that  $x_{ijkl}$  must be 0 for all  $ij$  when  $y_{kl} = 0$ . Constraint 3 specifies that each customer zone must be served by a single warehouse. Constraint 4 enforces the correct logical relationship between y and z and also keeps the total annual throughput between given maximum and minimum limits.

The author says that instead of using two sets of triply subscripted variables in two stages i.e. one from plant location to warehouse and other from warehouse to customer zones in the next stage, to use a quadruple subscripted formulation gives more flexibility. As the former suffers from the lack of flexibility for some applications because it “forgets” the origin of a commodity once it arrives at the warehouse which can be a serious drawback in real applications. Another advantage of the quadrupled subscripted formulation over the other style arises when some commodities are perishable. In such cases it may be necessary to disallow the possibility of shipping such commodities over  $jkl$  routes for which the total journey times are excessive. It also makes easy to accommodate direct plant-customer zone shipments in the proposed method.

Another unique feature of the proposed model is that no customer zone is allowed to deal with more than one warehouse, since  $y_{kl}$ 's must be 0 or 1 and not fractional. Thus each customers' demand must be satisfied by a single warehouse or directly from the plant.

Since most real life applications for the proposed formulation are too large at that to be solved economically by existing mixed integer programming codes the authors have

suggested the technique of Bender's Decomposition as the technique offers the possibility of making sequences of related runs in considerably reduced computing times as compared doing each run independently. The most striking conclusion from the work is that it can be seen from the results of the problems solved by using the proposed model. It is seen that small number of iterations are required for convergence even with very small values of optimality tolerances.

*Sharma (1991)* [2] has also used Benders' decomposition as a solution approach to solve a fertilizer production – distribution problem in Indian context; however it was used along with Lagrangean method also. According to the scope of the research work in the real life circumstances although the rail network in India is quite large, is not able to directly reach most of the fertilizer consumption centers. Therefore, in a typical distribution system the fertilizers are moved from the manufacturing plants to certain points known as rake points. From these rake points fertilizers are moved by road using heavy commercial vehicles to secondary points and finally reach the markets in light commercial vehicles. So essentially the formulation proposed is a two stage warehousing formulation with inventories also one of the factors included in the cost function. The decision making have to include in multi period context, at which rake and secondary points to locate warehouses, the size of the warehouses, the amount of inventory to be stocked at these warehouses, the transportation plan to meet the demand and how much to produce at each plant in each sub period. Severe restrictions exist on the quantities that can be transported in various directions as the number of rail wagons and HCV's are limited. The warehouse space available at various points is also limited.



In this paper the plants, rake points, secondary points and markets, will be referred to as points of type '1', '2', '3', '4' respectively.

The mathematical formulation of the above problem given by the author is give as below:

### *Constants of the Problem*

The cropping season of six months is divided into K number of periods (assumed even); H is the number of products and U denotes the number of nutrients. The quantity of nutrient demanded of type 'u' at market 'j' ( point of type 4) in period 'k' is denoted by  $D_{4jku}$  while  $f_{hu}$  is the fraction of nutrient of type 'u' obtainable from product of type 'h'. The total number of points of category 'i' is N (i); SN (i, j) is the set of points of type i+1 to which a point of type 'i' having a number 'j' can supply a fertilizer, RN (i, j) denotes the set of points of type i-1 from which a point of type 'i' having a number 'j' can receive fertilizer. Variables defined above have lower limit, upper limit and associated cost- coefficient and these constants are denoted by putting 'L', 'U' and 'C' respectively before their names.

Distribution cost is composed of production cost:  $CP_{1jkh} * P_{1jkh}$ ; cost of space  $CS_{1ij} * S_{1ij} + CS_{2ij} * S_{2ij}$ ; cost of carrying inventory  $0.5 * CI_{ijkh} * (E_{ijkh} + B_{ijkh})$ ; and the cost of locating a warehouse is  $CL_{ij} * L_{ij}$ .

### *Variable Definition*

The beginning and the ending inventory at point number 'j' of point type 'i' in the period 'k' of product type 'h' is represented by  $B_{ijkh}$ ,  $E_{ijkh}$  ;  $L_{ij}$  is the location variable which is 1 if it is decided to locate a warehouse at point number 'j' of point type 'i' (i=

2 to 3) and 0 otherwise. The quantity produced at plant number 'j'(point of type 1) in period k of product type 'h' is denoted by  $P_{1ijk}$ ;  $QTR_{4ijk}$  is the quantity received at market 'j'(point of type 4) in period 'k' of product type 'h'. The space booked at point number 'j' of point type 'i' in the first and last three months of a six monthly season is denoted by  $S1_{ij}$ ,  $S2_{ij}$  respectively. The total quantity transported from point number j1 of point type i1 to point number j2 of point type i2 in period 'k' of product type 'h' is represented by  $T_{i1j1,i2j2,k,h}$ ;  $TT_{ijk}$  denotes the total quantity transported from point number 'j' of point type 'i' in period 'k'.

So the objective function takes the following form including all the above components:

*MINIMIZE*

$$\sum_{jkh} CP_{1ijk} * P_{1ijk} + \sum_{ijkpmn} CT_{ijkpmn} * T_{ijkpmn} + \sum_{ijkh} 0.5 * CI_{ijkh} * (E_{ijkh} + B_{ijkh}) \\ + \sum_{ij} CS1_{ij} * S1_{ij} + \sum_{ij} CS2_{ij} * S2_{ij} + \sum_{ij} CL_{ij} * L_{ij}$$

*SUBJECT TO*

*Subproblem constraints*

$$P_{1jkh} + B_{1jkh} = E_{1jkh} + \sum_{m \in SN(1,j)} T_{1j2mkh} \quad (6)$$

for all  $j = 1, \dots, N(1)$ ;  $k = 1, \dots, K$ ;  $h = 1, \dots, H$

$$\sum_{m \in RN(i,j)} T_{(i-1)mijkh} + B_{ijkh} = E_{ijkh} + \sum_{m \in SN(i,j)} T_{ij(i+1)mkh} \quad (7)$$

for all 'i' = 2,...,3;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K$  and  $h = 1, \dots, H$

Lower and Upper Limit Constraints on Variables Defined (8)

$$QTR_{4 jkh} = \sum_{m \in RN(4, j)} T_{3m4 jkh} \quad (9)$$

For all  $j, k, h$

For the propose model there are  $K \cdot H$  number of sub problems and each sub problems have the constraints 1 to 4. 1 and 2 are material balance conditions at plant, rake points and the secondary storage points. 3 is to set lower and upper bounds on variables whereas 4 sees that demand is satisfied.

*Linking Constraints*

$$\sum_{h=1}^H f_{hu} * QTR_{4 jkh} \geq D_{4 jku} \quad (10)$$

For all  $u = 1, \dots, U$ ;  $j = 1, \dots, M$ ;  $k = 1, \dots, K$

$$B_{ij(k+1)h} = E_{ijkh} \quad (11)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K-1$  and  $h = 1, \dots, H$

$$TT_{ijk} = \sum_{h=1}^H \sum_{x \in SN(i, j)} T_{ij(i+1)xkh} \quad (12)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K$

$$\sum_{h=1}^H E_{ijkh} \leq S1_{ij} \quad (13)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K/2$

$$\sum_{h=1}^H E_{ijkh} \leq S2_{ij} \quad (14)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = K/2+1, \dots, K$

*Location constraint*

$$L_{ij} = (0, 1) \text{ for all } i, j \quad (15)$$

Constraint (16) gives a set of linear constraints representing the condition that if the warehouse is located at a particular point then the space booked must be within the permissible upper and lower limits

Constraints (17) are the non negativity constraints of all real variables

The author has used the application of Bender's Decomposition to solve the above defined problem. The primal problem was attempted by two Lagrangean relaxation procedures.

The use of the two relaxations is given as below in which a single product and a single nutrient is considered for purpose of illustration.

#### *Relaxation 1:LR1*

The constraints of the type  $Bx = b$  in the above formulation are added to the objective function with the multipliers.

$$B_{ij(k+1)h} = E_{ijkh} \quad (11)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K-1$  and  $h = 1, \dots, H$

$$\sum_{h=1}^H E_{ijkh} \leq S1_{ij} \quad (13)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K/2$

$$\sum_{h=1}^H E_{ijkh} \leq S2_{ij} \quad (14)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = K/2+1, \dots, K$

Multipliers for the equations 6, 8, 9 are  $u_{ijk}$ ,  $v_{ijk}$ ,  $w_{ijk}$  respectively. Starting values of these multipliers are chosen as follows

$u_{ijk} = 0$  for  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K-1$

$v_{ijk}, w_{ijk} = \min (CS1_{ij}, CS2_{ij})$  for  $i = 2, 3$ ;  $j = 1, \dots, N(i)$ ; and all  $k$

## Relaxation 2:LR2

New constraints were prepared as follows.

$$B_{ij(k+1)h} = E_{ijkh} \quad (11)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K-1$  and  $h = 1, \dots, H$

$$\sum_{h=1}^H E_{ijkh} \leq S1_{ij} \quad (13)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K/2$

$$\sum_{h=1}^H E_{ijkh} \leq S2_{ij} \quad (14)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = K/2+1, \dots, K$

$$B_{ij(k+1)l} \leq S1_{ij} \quad (18)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K/2$

$$B_{ij(k+1)l} \leq S2_{ij} \quad (19)$$

For  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = K/2, \dots, K-1$

Multipliers for the above equations are denoted by  $u_{ijk}$ ,  $v1_{ijk}$ ,  $v2_{ijk}$ ,  $w1_{ijk}$ ,  $w2_{ijk}$  respectively.

Starting values for the multipliers were chosen as follows:

$u_{ijk} = 0$  for  $i = 1, 2, 3$ ;  $j = 1, \dots, N(i)$ ;  $k = 1, \dots, K-1$

$v1_{ijk}, w1_{ijk}, v2_{ijk}, w2_{ijk} = 0.5 * \min (CS1_{ij}, CS2_{ij})$  for  $i = 2, 3$ ;  $j = 1, \dots, N(i)$ ; and all  $k$

The Lagrangean relaxation procedure is stopped when the sum of total booked warehouse space attains the minimum level to be able to meet the demand. A backward recursive procedure is then used to get a feasible primal solution. That is, for the relative cost coefficients that are passed down to the sub problems, the last period ( $K$ ) is solved first and its beginning inventories are passed down to previous period ( $K-1$ ) problem as

ending inventories, and then problem of period (K-1) is solved, and again its beginning inventories are passed down to period (K-2). This recursive procedure is followed till the first problem is solved. Finally, the warehouse space booked is computed by setting it to the maximum of the ending inventory.

It was found that, on an average for problems of different sizes LR2 converged in around one third the number of iterations taken by LR1. In LR2 the relative cost coefficient of warehouse space booked is a function of multipliers which determine the relative cost coefficient of beginning and ending inventories, whereas in LR1 the relative cost coefficient of the warehouse space booked is a function of multipliers that determine the relative cost coefficient of the ending inventories only. This could result in the better performance of LR2 than LR1.

So the model developed here serves as a good aid to making decisions and routine sensitivity analysis is possible as complete information on dual variables is available.

## **2.4 Lagrangean Relaxation**

*Sharma (1996)* [4] presents a mixed integer linear programming formulation as faced by the Food Corporation of India (FCI) which turns out to be single stage warehouse location problem (SSWLP), and suggest a branch and bound based procedure for the solution. At each node the branch and bound procedure results a minimum cost flow problem which can be solved in  $O(n^2 \ln(n))$  time by using best known procedures. The paper presents a Lagrangean based heuristic procedure which obtains a good bound for the associated minimum cost flow problem in  $O(n^2)$  time thus offering substantial saving in computations which become particularly significant while solving a large sized real life problem as faced by the FCI.

After giving a mixed 0-1 integer linear programming formulation of the single stage warehouse location problem the author relaxes the integer constraints in the problem to obtain the linear programming relaxation of the problem. In the linear programming relaxation of the problem further few constraints are relaxed and it is shown that the reduced problem can be solved optimally by an efficient heuristic. Also profitable use of the same method is used in Lagrangean based solution procedure to obtain a good bound to the optimal solution value of the linear programming relaxation. The optimal integer solution to the problem is obtained by initiating a branch and bound solution procedure.

#### *Problem Formulation*

In the formulation plants, trans-shipment points and the markets are denoted by points of type '1', '2' and '3' respectively

#### *Constants of the problem*

$S_j$  is the maximum quantity that can be supplied by a plant 'j';  $D_k$  is the demand at market 'k';  $f_i$  is the fixed cost of locating a trans-shipment point;  $c_{ij}$  is the cost of transporting  $\sum D_k$  units of goods from point number 'j' of type 1 to point number 'i' of type 2;  $B_{ik}$  is the cost of transporting  $\sum D_k$  units of goods from point number 'i' of type 2 to point number 'k' of type 3.

#### *Variable Definition*

$Y_{ij}$  is the quantity received (as a fraction of the total demand) by the point number 'i' of type 2 from point number 'j' of type 1 ;  $Y_i$  is the location variable which is 1 if it is decided to locate a warehouse at point number 'i' of type 2, 0 otherwise.  $X_{ik}$  is the

fraction of demand at point number 'k' of type 3 which is supplied by point 'i' of type 2.

MINIMIZE

$$\sum_j \sum_i Y_{ij} \cdot C_{ij} + \sum_i f_i Y_i + \sum_k \sum_i X_{ik} \cdot B_{ik}$$

SUBJECT TO

$$\sum_j \sum_i Y_{ij} = 1 \quad (20)$$

$$\sum_i Y_{ij} \leq S_j / (\sum_k D_k) \forall j \quad (21)$$

$$Y_i \geq \sum_j Y_{ij} \forall i \quad (22)$$

$$\sum_i X_{ik} = 1 \forall k \quad (23)$$

$$X_{ik} \leq Y_i \forall i, k \quad (24)$$

$$\sum_j Y_{ij} = \sum_k X_{ik} (D_k / \sum_k D_k) \forall i \quad (25)$$

$$Y_{ij} \geq 0 \forall i, j \quad (26)$$

$$X_{ik} \geq 0 \forall i, k \quad (27)$$

$$Y_i = (0, 1) \forall i \quad (28)$$

Constraint 20 ensures that adequate quantities are shipped from plants to warehouses so that total demand can be met. Equation 21 allows plants to supply within their capacities, 22 ensures that a warehouse is located only when the quantity shipped to the warehouse point is greater than zero. Equation 23 is for meeting the demand at each market point and 24 ensures that the warehouse is located only when the outflow from



the same is positive. Equation 25 are the flow balance constraints at each warehouse point i.e. inflow at a warehouse point from the plants is equal to the outflow to the markets. 26 and 27 are non negativity constraint whereas 28 is the integer constraint.

Next the author relaxes constraint 25 and the resulting linear programming formulation is as follows.

MINIMIZE

$$\sum_j \sum_i Y_{ij}.C_{ij} + \sum_i f_i Y_i + \sum_k \sum_i X_{ik}.B_{ik}$$

Subject to

20 to 28 and  $Y_i \geq 0$  for all 'i'.

The new problem is clearly is the minimum cost flow network problem for which best known polynomial algorithms  $O(n^2 \ln(n))$  are available. Now the author associates multipliers with equations 24 and 25 and include them in the objective function. And then he goes on to show that the reduced problem is solvable in  $O(n^2)$  time.

The main problem so gets decomposed into two problems as below:

MINIMIZE

$$\sum_j \left[ \sum_i Y_{ij}(C_{ij} + \mu_i) \right] + f * \sum_i (f_i - \sum_k \lambda_{ik}) Y_i \quad (29)$$

Subject to

20 to 22 and 26

Where  $0 \leq f \leq 1$ ,  $f$  is a constant and known

And

MINIMIZE

$$(1 - f) * \sum_i (f_i - \sum_k \lambda_{ik}) Y_i + \quad (30)$$

$$\sum_k \left[ \sum_i X_{ik} (B_{ik} + \lambda_{ik} - \mu_{ik} D_k / \sum_k D_k) \right]$$

Subject to

23 and 27

Where  $0 \leq f \leq 1$ ,  $f$  is a constant and known

Fraction  $f$  of the fixed costs is included in the first problem whereas the remaining portion of the fixed costs is included in the second part.

The author then gives heuristics to solve the two parts of the problem and it is shown that the optimal solutions for these problems can be obtained in  $O(n^2)$  time.

## 2.5 Simulation Based Approach

*Hidaka, Okano (1997)* [5] have used simulation for large-scale Warehouse (Facility) Location Problems in the real world on a digital map, and found an approximate solution. The problem was to find a near-optimal solution for the number and locations of warehouses that minimize the sum of the transportation cost and fixed cost, and meet the needs of 6,800 customers. The network data of the digital map were efficiently used to obtain candidate warehouse locations, to simulate the transportation cost, to simulate the warehouse fixed cost, and to find a near-optimal solution for the number and locations of warehouses.

In order to solve a real instance of a large-scale warehouse location, they developed simulation-based methods. That is, they simulated the warehouse location problem in the real world on a digital map.

First, all warehouses and customers were assumed to be located at nodes of a network covering the whole given area. From the customer nodes, sets of candidate warehouse locations were selected, and the fixed and transportation costs for each candidate were calculated on the basis of the real data for the current 11 warehouses. The transportation cost was assumed to be proportional to the delivery time.

Next, they optimized the number and locations of the warehouses in two steps. The first step was to apply Greedy – Interchange heuristics with the warehouse candidates selected in the above manner. Using these heuristics, they found an approximately optimal solution for the number and locations of warehouses selected from the warehouse candidates.

Next, in order to improve the solution obtained in the first step, they devised a heuristic procedure for finding medians, and named it “Balloon Search.” This procedure may be outlined as follows: they relocated each warehouse obtained in the first-step solution to an adjacent node of the network that was not selected as a warehouse candidate node. Warehouse ‘i’ was relocated by aiming to find one median of the corresponding sub network that included only the subset of customers who were assigned to warehouse ‘i’. In this process, they first reduced the subset of customers so that it included only customers whose transportation costs were relatively low. Then, they increased the number of the customers in the subset step by step, and repeatedly found medians. During these processes, they assumed that the fixed cost of the adjacent node was the same as that of the warehouse obtained in the first-step. After the first and second optimization steps, a near-optimal solution for the number and locations of warehouses was expected to be found from the large number of network nodes of the digital map.

## 2.6 Discussion

Geoffrion and Graves (1974) [1] and Sharma (1991) [2] have given different styles of formulations of SSCWLP. We use these formulation styles and add a variety of linking constraints (that link binary 0-1 variable or location variable to real variables that denote quantity shipped from plant to market through warehouse). This results in variety of different formulations of SSCWLP. We relax the integer 0-1 constraint in these formulations to obtain various relaxations. These are given in chapter 3. Later we carryout an empirical investigation to determine strengths of various relaxations. This is done in chapter 4.

## Chapter 3

### New formulations for the SSCWLP

In this chapter we will enlist all the new formulations of SSCWLP. In the Chapter 2 during the literature review we see that there are two major styles of mathematical model formulation for the SSWLP. In the first style the whole distribution of commodity from plant to markets is covered in a single phase as done by *Geoffrion and Graves (1974)* [1]. The other style is a variation of this as used by *Sharma (1991)* [2] in which he uses a two phase for distribution of commodities (first from the plant to the warehouse and then from the warehouse to the markets).

So in this chapter also we have used both the styles and called the first style as STYLE1 and the latter as STYLE2. In the following section we will list our formulations for both the styles one by one with all the relaxations. In each style there are two versions of all the formulations which are given in the chapter. For the purpose of illustration; only one commodity is considered.

#### **3.1 Problem formulation (STYLE 1 and Variation 1)**

In this section we will enlist the various formulations of style 1 as adopted by *Geoffrion and Graves (1974)* [1].

##### **3.1.1 Constants Definition**

$D_k$	Demand for the commodity at market 'k'
$d_k$	$D_k / \sum D_k$ demand at market 'k' as a fraction of total market demand.
$S_i$	Supply available at plant 'i' of the commodity
$s_i$	$S_i / \sum D_k$ supply available at plant 'i' as a fraction of the total market demand

$f_j$	Fixed cost of locating a warehouse at 'j'
$C_{ijk}$	Cost of transporting $\sum D_k$ quantity of goods from 'i' to 'j' to market 'k'
$CAP_j$	Capacity of a warehouse 'j'
$cap_j$	$CAP_j / \sum D_k$ capacity of a warehouse at 'j' as a fraction of the total market demand

### 3.1.2 Variable Definition

$X_{ijk}$	quantity of commodity transported from plant 'i', to warehouse 'j' to market 'k'
$x_{ijk}$	$X_{ijk} / \sum D_k$ quantity transported as a fraction of total market demand
$y_j$	1 if warehouse is located at 'j', 0 otherwise

### 3.1.3 Mathematical Formulation

Now cost minimization problem for the SSCWLP can be written as the mixed integer programming problem as follows

#### FORMULATION S1V1.1(A)

$$\text{MIN} \quad \sum_i \sum_j \sum_k C_{ijk} . x_{ijk} + \sum_j f_j . y_j \quad (0)$$

Subject to----

$$\sum_i \sum_j \sum_k x_{ijk} = 1 \forall i, j, k \quad (1)$$

$$\sum_j \sum_k x_{ijk} \leq s_i \forall i \quad (2)$$

$$\sum_i \sum_j x_{ijk} \geq d_k \forall k \quad (3)$$

$$x_{ijk} \geq 0 \forall i, j, k \quad (4)$$

$$\sum_i \sum_k x_{ijk} \leq cap_j y_j \forall j \quad (5)$$

$$\sum_i \sum_k x_{ijk} \leq y_j \forall j \quad (6)$$

$$y_j = (0, 1) \forall j \quad (7)$$

So the optimal integer solution to the problem is obtained by solving the above mixed integer formulation.

MINIMIZE (0) subject to (1) to (7)

The first part of the objective function denotes the cost of transportation of the commodity from the plant to the market via the warehouse; the second part is the fixed cost of locating the warehouse.

Constraint 1 ensures that demand is met at each market point. 2 ensure that plants supply within their supplying capacities; 3 ensures that the demand at each point is met, whereas 4 is the non-negativity constraint. Constraint 5 ensures that the warehouses are routing the commodity to the markets from the plants within their capacity which is known; equation 6 is the linking constraint and ensures that warehouse is located only if the quantity shipped from the warehouse point is greater than zero. Equation 7 is the integer constraint on  $y_i$ .

Now we relax the constraint 7 and so the problem now become

#### FORMULATION S1V1.1(B)

MINIMIZE (0) subject to (1) to (6) and (8) where

$$y_j \geq 0 \forall j \quad (8)$$

This can be referred as the weak relaxation as termed so in the literature (*Krarup and Purzan, (1983) [12]; Erlenkotter (1978) [14]* as it is on the similar lines for weak relaxations of SPLP (Simple Plant Location Problem); and for CPLP (Capacitated Plant Location Problem) as given in *Cornuejols et.al. (1991) [8]*.

In the next few variation of the model constraint (6) is relaxed and replaced by other linking constraints.

#### FORMULATION S1V1.2(B)

So when we replace the constraint (6) and replace it by (9) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8) and (9)

$$\begin{aligned} \sum_i \sum_k x_{ijk} + M(1 - y_j) &\geq 0 \forall j \\ \sum_i \sum_k x_{ijk} + M y_j &\geq 0 \forall j \\ \sum_i \sum_k x_{ijk} - M y_j &\leq 0 \forall j \end{aligned} \quad (9)$$

Equation (9) is another form of the linking constraint and ensures that the warehouse is located only when there is quantity shipped through the warehouse is greater than zero.



We call this constraint the Big M constraint where M is a constant having a very large value.

#### FORMULATION S1V1.3(B)

So when we replace the constraint (6) and replace it by (10) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8) and (10)

$$\sum_i x_{ijk} \leq y_j d_k \forall j, k \quad (10)$$

Equation (10) is another form of linking constraint and also ensures that the warehouse is located only if quantity shipped through the warehouse is greater than zero. In the literature the above relaxation is known as the strong relaxation.

#### FORMULATION S1V1.4(B)

So when we replace the constraint (6) and replace it by (11) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8) and (11)

$$\begin{aligned} \sum_i x_{ijk} - M(1 - y_j) &\leq d_k \forall j, k \\ \sum_i x_{ijk} - M y_j &\leq 0 \forall j, k \\ \sum_i x_{ijk} + M y_j &\geq 0 \forall j, k \end{aligned} \quad (11)$$

Equation (11) is another form of linking constraint and also ensures that the warehouse is located only if quantity shipped through the warehouse is greater than zero. This constraint is another variation of the Big M constraint where M is a constant having a very large value.

#### FORMULATION S1V1.5(B)

So when we replace the constraint (6) and replace it by (10) and (12) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8) and (10) and (12)

$$\sum_k x_{ijk} \leq y_j s_i \forall i, j \quad (12)$$

Equation (10) and (12) are another form of linking constraints and also ensures that the warehouse is located only if quantity shipped through the warehouse is greater than zero. They relate the quantity shipped from a plant 'i' to the market 'k' through warehouse 'j' to the supply from the plant and the demand at the market. This is another form of a strong relaxation.

#### FORMULATION S1V1.6(B)

So when we replace the constraint (6) and replace it by (11) and (13) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8) and (11) and (13)

$$\begin{aligned}
\sum_k x_{ijk} - M(1 - y_j) &\leq s_i \forall i, j \\
\sum_k x_{ijk} - My_j &\leq 0 \forall i, j \\
\sum_k x_{ijk} + My_j &\geq 0 \forall i, j
\end{aligned} \tag{13}$$

Equation (11) and (13) are another form of linking constraint and also ensures that the warehouse is located only if quantity shipped through the warehouse is greater than zero. They relate the quantity shipped from a plant 'i' to the market 'k' through warehouse 'j' to the supply from the plant and the demand at the market. This is another form of a Big M relaxation.

#### FORMULATION S1V1.7(B)

So when we totally remove the linking constraint (6) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (5), (8)

This is done to see the bounds given by the problem without the linking constraint and to see the strength of  $y_j$  in the constraint (5).

For all the above formulations S1V1.2(B) to S1V1.7(B) we have the problems giving the optimal integer solution just by replacing the constraint (8) with constraint (7) thus giving formulations S1V1.2(A) to S1V1.7(A). At the stage of computational experience the integer formulation are used to verify that all the formulations give the same optimal results.

### 3.2 Problem formulation (STYLE 1 and Variation 2)

In this section we will enlist all the formulation with Style 1 same as the style in the above section but only variation being in constraint (5). In this variation all the problems (from 1(A) to 6(B)) have the identical structures except that the constraint (5) is added in a different form as given below.

$$\sum_i \sum_k x_{ijk} \leq cap_j \forall j \longrightarrow (14)$$

So the integer formulation for the variation 2 is as follows:

#### FORMULATION S1V2.1(A)

MINIMIZE (0) subject to (1) to (4),(6) to (7) and (14)

Now we relax the constraint 7 and so the problem now becomes

#### FORMULATION S1V2.1(B)

MINIMIZE (0) subject to (1) to (4), (6), (8) and (14)

In the next few variation of the model constraint (6) is relaxed and replaced by other linking constraints.

#### FORMULATION S1V2.2(B)

So when we replace the constraint (6) and replace it by (9) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (9) and (14)

#### FORMULATION S1V2.3(B)

So when we replace the constraint (6) and replace it by (10) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (10) and (14)

In the literature the above relaxation is known as the strong relaxation.

#### FORMULATION S1V2.4(B)

So when we replace the constraint (6) and replace it by (11) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (11) and (14)

#### FORMULATION S1V2.5(B)

So when we replace the constraint (6) and replace it by (10) and (12) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (10), (12) and (14)

#### FORMULATION S1V2.6(B)

So when we replace the constraint (6) and replace it by (11) and (13) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (11), (13) and (14)

For all the above formulations S1V2.2(B) to S1V2.6(B) we have the problems giving the optimal integer solution just by replacing the constraint (8) with constraint (7) thus giving formulations S1V2.2(A) to S1V2.6(A). At the stage of computational experience

the integer formulation are used to verify that all the formulations give the same optimal results.

### 3.3 Problem formulation (STYLE 2 and Variation 1)

In this section we will enlist the various formulations of style 2 as adopted by *Sharma (1991)* [2]. Here the transportation of the commodity is done in two stages. In first stage the commodity is considered to be transferred from plant to the warehouse and then in next stage from the warehouse to the market.

#### 3.3.1 Constants Definition

$D_k$	Demand for the commodity at market 'k'
$d_k$	$D_k/\sum D_k$ demand at market 'k' as a fraction of total market demand.
$S_i$	Supply available at plant 'i' of the commodity
$s_i$	$S_i/\sum D_k$ supply available at plant 'i' as a fraction of the total market demand
$f_j$	Fixed cost of locating a warehouse at 'j'
$C_{pwi}$	Unit cost of transporting goods from plant 'i' to warehouse 'j'
$C_{wmjk}$	Unit cost of transporting goods from warehouse 'j' to market 'k'
$CAP_j$	Capacity of a warehouse 'j'
$cap_j$	$CAP_j/\sum D_k$ capacity of a warehouse at 'j' as a fraction of the total market demand

#### 3.3.2 Variable Definition

$X_{pwi}$	Quantity of commodity transported from plant 'i', to warehouse 'j'.
$x_{pwi}$	$X_{pwi}/\sum D_k$ quantity transported as a fraction of total market demand.
$X_{wmjk}$	Quantity of commodity transported from warehouse 'j' to market 'k'.
$x_{wmjk}$	$X_{wmjk}/\sum D_k$ quantity transported as a fraction of total market demand.
$y_j$	1 if warehouse is located at 'j', 0 otherwise

### 3.3.3 Mathematical Formulation

Now cost minimization problem for the SSCWLP can be written as the mixed integer programming problem as follows

#### FORMULATION S2V1.1(A)

$$\text{MIN} \sum_i \sum_j x_{pwij} . C_{pwij} + \sum_j \sum_k x_{wmjk} . C_{wmjk} + \sum_j f_j y_j \quad (00)$$

SUBJECT TO -----

$$\sum_j x_{pwij} \leq s_i \forall i \quad (15)$$

$$\sum_j x_{wmjk} \geq d_k \forall k \quad (16)$$

$$\sum_i \sum_j x_{pwij} = 1 \quad (17)$$

$$\sum_j \sum_k x_{wmjk} = 1 \quad (18)$$

$$x_{pwij} \geq 0 \forall i, j \quad (19)$$

$$x_{wmjk} \geq 0 \forall j, k \quad (20)$$

$$\sum_i x_{pwij} = \sum_k x_{wmjk} \forall j \quad (21)$$

$$\sum_i x_{pwij} \leq cap_j y_j \forall j \quad (22)$$

$$\sum_i x_{pwij} \leq y_j \forall j \quad (23)$$

$$y_j = (0,1) \forall j \quad (24)$$

So the optimal integer solution to the problem is obtained by solving the above mixed integer formulation.

MINIMIZE (00) subject to (15) to (24)

The first part of the objective function denotes the cost of transportation of the commodity from the plant to the warehouse the second part is the cost of transportation of the commodity from the warehouse to the market; and the third part is the fixed cost of locating the warehouse.

Constraint 15 ensures that plants supply within their supplying capacities; 16 ensures that the demand at each point is met, whereas 17 and 18 ensure that the demand is met at each point. Constraints 19 and 20 are the non-negativity constraint. Constraint 21 is the flow balance constraint and ensures that the commodity coming into the warehouse is passed to the markets. Constraint 22 ensures that the warehouses are routing the commodity to the markets from the plants within their capacity which is known; equation 23 is the linking constraint and ensures that warehouse is located only if the quantity shipped from the warehouse point is greater than zero. Equation 24 is the integer constraint on  $y_i$ .

Now we relax the constraint 24 and so the problem now becomes

#### FORMULATION S2V1.1(B)

MINIMIZE (00) subject to (15) to (23) and (25) where

$$y_j \geq 0 \forall j \quad (25)$$



This can be referred as the weak relaxation as termed so in the literature (*Krarup and Purzan, (1983) [12]; Erlenkotter (1978) [14]* as it is on the similar lines for weak relaxations of SPLP (Simple Plant Location Problem); and for CPLP (Capacitated Plant Location Problem) as given in *Cornuejols et.al. (1991) [8]*.

In the next few formulations of the model constraint (23) is relaxed and replaced by other linking constraints.

#### FORMULATION S2V1.2(B)

When we replace the constraint (23) and replace it by (26) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22),(25) and (26)

$$\begin{aligned} \sum_i x_{pwi j} + M(1 - y_j) &\geq 0 \forall j \\ \sum_i x_{pwi j} + My_j &\geq 0 \forall j \\ \sum_i x_{pwi j} - My_j &\leq 0 \forall j \end{aligned} \tag{26}$$

Equation (26) is another form of the linking constraint and ensures that the warehouse is located only when there is quantity shipped to the warehouse is greater than zero. We call this constraint the Big M constraint where M is a constant having a very large value.

#### FORMULATION S2V1.3(B)

So when we replace the constraint (23) and replace it by (27) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22), (25) and (27)

$$x_{wmjk} \leq y_j d_k \forall j, k \quad (27)$$

Equation (27) is another form of linking constraint and also ensures that the warehouse is located only if quantity shipped from the warehouse is greater than zero. It links the demand at market 'k' and the quantity to be transported. In the literature the above relaxation is known as the strong relaxation.

#### FORMULATION S2V1.4(B)

So when we replace the constraint (23) and replace it by (28) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22), (25) and (28)

$$\begin{aligned} x_{wmjk} - M(1 - y_j) &\leq d_k \forall j, k \\ x_{wmjk} - My_j &\leq 0 \forall j, k \\ x_{wmjk} + My_j &\geq 0 \forall j, k \end{aligned} \quad (28)$$

Equation (28) is another form of linking constraint and also ensures that the warehouse is located only if quantity shipped from the warehouse is greater than zero. This formulation is another variation of the Big M constraint where M is a constant having a very large value.

#### FORMULATION S2V1.5(B)

So when we replace the constraint (23) and replace it by (27) and (29) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22), (25) and (27) and (29)

$$x_{pwij} \leq y_j s_i \forall i, j \quad (29)$$

Equation (27) and (29) are another form of linking constraint and also ensures that the warehouse is located only if quantity shipped through the warehouse is greater than zero. They relate the quantity shipped from a plant 'i' to the market 'k' through warehouse 'j' to the supply from the plant and the demand at the market. This is another form of a strong relaxation.

#### FORMULATION S2V1.6(B)

When we replace the constraint (23) and replace it by (28) and (30) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22), (25) and (28) and (30)

$$\begin{aligned} x_{pwij} - M(1 - y_j) &\leq s_i \forall i, j \\ x_{pwij} - My_j &\leq 0 \forall i, j \\ x_{pwij} + My_j &\geq 0 \end{aligned} \quad (30)$$

Equation (28) and (30) are another form of linking constraint and also ensures that the warehouse is located only if quantity shipped to and from the warehouse is greater than zero. They relate the quantity shipped from a plant 'i' to the market 'k' through

warehouse 'j' to the supply from the plant and the demand at the market. This is another form of a Big M relaxation.

#### FORMULATION S2V1.7(B)

So when we totally remove the linking constraint (23) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (22), (25)

This is done to see the bounds given by the problem without the linking constraint and to see the strength of  $y_j$  in the constraint (8).

For all the above formulations S2V1.2(B) to S2V1.7(B) we have the problems giving the optimal integer solution just by replacing the constraint (25) with constraint (24) thus giving formulations S2V1.2(A) to S2V1.7(A). At the stage of computational experience the integer formulation are used to verify that all the formulations give the same optimal results.

### **3.4 Problem formulation (STYLE 2 and Variation 2)**

In this section we will enlist all the formulation with Style 2 same as the style in the above section but only variation being in constraint (22). In this variation all the problems (from 1(A) to 6(B)) have the identical structures except that the constraint (22) is added in a different form as given below.

$$\sum_i x_{pij} \leq cap_j \forall j \quad (31)$$

So the integer formulation for the variation 2 is as follows:

FORMULATION S2V2.1(A)

MINIMIZE (00) subject to (15) to (21),(23) to (24) and (31)

Now we relax the constraint 24 and so the problem now becomes

FORMULATION S2V2.1(B)

MINIMIZE (00) subject to (15) to (21),(23), (25) and (31) where

In the next few formulations of the model constraint (23) is relaxed and replaced by other linking constraints.

FORMULATION S2V22(B)

When we replace the constraint (23) and replace it by (26) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (21),(25), (26) and (31)

FORMULATION S2V2.3(B)

So when we replace the constraint (23) and replace it by (27) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (21), (25), (27) and (31)

FORMULATION S2V2.4(B)

So when we replace the constraint (23) and replace it by (28) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (21), (25), (28) and (31)

#### FORMULATION S2V2.5(B)

So when we replace the constraint (23) and replace it by (27) and (29) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (21), (25), (27), (29) and (31)

#### FORMULATION S2V2.6(B)

When we replace the constraint (23) and replace it by (28) and (30) as given below the problem becomes

MINIMIZE (00)

Subject to (15) to (21), (25), (28), (30) and (31)

For all the above formulations S2V2.2(B) to S2V2.6(B) we have the problems giving the optimal integer solution just by replacing the constraint (25) with constraint (24) thus giving formulations S2V2.2(A) to S2V2.6(A). At the stage of computational experience the integer formulation are used to verify that all the formulations give the same optimal results.

Hence in this chapter we enlist two styles of formulations with their two variations and with all possible relaxations.

### 3.5 Summary

In this section we give the summary of the formulations given in this chapter and give the nomenclature to be used in the subsequent sections:

For both the Styles and Variations the formulations can be summarized as below

- \*.1B: Weak relaxation of SSCWLP
- \*.3B: SRS of one kind ( $y_j d_k \geq \sum_i x_{ijk}$ )
- \*.5B: SRS of another kind ( $y_j d_k \geq \sum_i x_{ijk}$  &  $y_j s_i \geq \sum_k x_{ijk}$ )
- \*.2B: Big-M of first kind
- \*.4B: Big-M of another kind
- \*.6B: Two Big-M relaxations combined

Variation 1:  $\text{cap}_j y_j \geq \sum_{ik} x_{ijk}$

Variation 2:  $\text{cap}_j \geq \sum_{ik} x_{ijk}$

## Chapter 4

### STRUCTURE OF EMPIRICAL INVESTIGATION CARRIED

#### 4.1 Introduction

We have tested the two styles of formulation first given by Geoffrion and Graves (1974) [1] and the second one used by Sharma (1991) [2]. For each of these styles we have two variations; the first one having a stronger relaxation for the capacity constraint and the other one having not so strong relaxation for the capacity constraint. For all these styles and variations there are seven different formulations in each category obtained by relaxing the linking constraint as listed down in chapter 3.

We have tested the various relaxations for solving SSCWLP on a large variety of problems so as to see the effect of various parameters on the results. We have tried both small and large size problems for the warehouse location model and in this chapter we give the computational results as obtained by solving the problems. We have used the optimization package LINGO for solving the problems for various relaxations of SSCWLP. All the problems have been randomly generated and the results for the objective values and the number of iterations have been listed down.

The problem testing phase has included the problems to be tested in the following manner.



## 4.2 Computational Details

**TABLE 4.1(PROBLEMS FOR PROBLEM SIZE 5X5X5)**

Problem size	Style type	Variation type	Capacity	Supply	Relaxation	No of problems
5X5X5	Style 1	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 1	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 2	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 2	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 1	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 1	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 2	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
5X5X5	Style 2	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
<b>TOTAL NO. OF PROBLEMS</b>						<b>720</b>

The Problem size 5X5X5 means that there are 5 plants, 5 warehouses to choose from and 5 demand sites.

Similarly problems of size 10X10X10 and 20X20X20 are tried.

**TABLE 4.2(PROBLEMS FOR PROBLEM SIZE 10X10X10)**

Problem size	Style type	Variation type	Capacity	Supply	Relaxation	No of problems
10X10X10	Style 1	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 1	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 2	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 2	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 1	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90

10X10X10	Style 1	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 2	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
10X10X10	Style 2	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
<b>TOTAL NO. OF PROBLEMS</b>						<b>720</b>

**TABLE 4.3(PROBLEMS FOR PROBLEM SIZE 20X20X20)**

<b>Problem size</b>	<b>Style type</b>	<b>Variation type</b>	<b>Capacity</b>	<b>Supply</b>	<b>Relaxation</b>	<b>No of problems</b>
20X20X20	Style 1	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 1	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 2	Variation1	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 2	Variation2	$\Sigma cap_j = 2.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 1	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 1	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 2	Variation1	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
20X20X20	Style 2	Variation2	$\Sigma cap_j = 6.0$	$\Sigma s_i = 1, 1.5, 2, 5, 10, 20, 30, 40, 50$	For each relaxation	90
<b>TOTAL NO. OF PROBLEMS</b>						<b>720</b>

So in all 2160 different problems were tried to computationally establish the results and to have an effective analysis at later stages.

Problems of bigger sizes could not be tried as the optimization package LINGO available in the labs is an educational package and has limitations on the number of variables and constraints in a problem that could be tried on the package.

### 4.3 Results

In Appendix 1 the results obtained during the solving of various size problems ranging from 5X5X5 to 20X20X20 are given for various formulations. The objective value obtained by solving each of the problem and the number of iterations taken is shown.

## Chapter 5

### ANALYSIS OF DATA AND RESULTS

#### 5.1 Introduction

In this chapter statistical analysis of the data obtained in chapter 4 is done so as to bring forward some conclusive results from the work. The organization of the chapter is as follows.

In section 5.2 we will list down the results for the main findings of the work; that the results obtained by Big M formulations (S1V1.2B, S1V1.4B, S1V1.6B) give statistically significant bounds obtained by other relaxations (S1V1.1B, S1V1.3B, S1V1.5B) for problems with  $\Sigma cap_j = 2.0$  and for both the styles. Also in this section findings for case  $\Sigma cap_j = 6.0$  is included. In section 5.3 we give the analysis results to show that the bounds obtained by S1V1.1B, S1V1.3B, S1V1.5B are same and have no statistically significant difference. In section 5.4 we will list down the results of analysis proving that the bounds obtained by Variation 1 for each problem category has better bounds than obtained by Variation 2. In section 5.5 we will give the results showing that in most circumstances Style 2 gives the same bounds as Style1 with less number of iterations that is Style 2 is computationally less expensive than Style1. Conclusions are listed down after each of the results establishing it. In the following sections all the T-tests and F-test have been done considering the null hypothesis that the difference between the means is zero.

### 5.2.1 Statistical Significance of Big M Formulations (Case $\Sigma \text{cap}_j = 2.0$ )

This section deals with the most important finding of the work that when  $\Sigma \text{cap}_j = 2.0$  then for both the styles and for Variation 1 the Big M formulations (S1V1.2B, S1V1.4B, S1V1.6B, S2V1.2B, S2V1.4B, S2V1.6B) give the same bounds which are better than the bounds given by other relaxations (S1V1.1B, S1V1.3B, S1V1.5B, S2V1.1B, S2V1.3B, S2V1.5B). Since the bounds obtained by S1V1.1B, S1V1.3B, S1V1.5B, and S2V1.1B, S2V1.3B, S2V1.5B are same with no statistically significant difference we have carried out the T-test on S1V1.1B and S1V1.2B and also on S2V1.1B and S2V1.2B so as to establish that Bound (.2B) are significantly greater than Bound (.1B)

Hence establishing that

Bound (.2B, .4B, .6B) are significantly greater than Bound (.1B, .3B, .5B)

The results of the T-test is given in

**TABLE 5.1(T-test between .1B's and .2B's)**

Problem description	T calculated
5X5X5 S1V1.1B AND S1V1.2B	-14.08
5X5X5 S2V1.1B AND S2V1.2B	-14.10
5X5X5 S1V2.1B AND S1V2.2B	77.67
5X5X5 S2V2.1B AND S2V2.2B	77.66
10X10X10 S1V1.1B AND S1V1.2B	-16.59
10X10X10 S2V1.1B AND S2V1.2B	-16.59
10X10X10 S1V2.1B AND S1V2.2B	95.68
10X10X10 S2V2.1B AND S2V2.2B	95.74
20X20X20 S1V1.1B AND S1V1.2B	-24.31
20X20X20 S2V1.1B AND S2V1.2B	-24.31
20X20X20 S1V2.1B AND S1V2.2B	125.63
20X20X20 S2V2.1B AND S2V2.2B	125.64

The above values for T-calculated can be compared with the following values of T-critical at the different significance levels (for set of 90 data).

**Table 5.2 (T-critical values data size 90)**

Significance level	Alpha= 0.05	Alpha= 0.01	Alpha= 0.005
T-critical(one tail)	1.662156	2.368979	2.632205
T-critical(two tailed)	1.986978	2.632205	2.878696

To support the fact that the Big M formulations give significantly better bounds is strengthened by the table 5.3 given below indicating the % improvement in the bounds given by Big M formulations over other relaxations.

**Table 5.3 (% Improvement of Big-M formulation over SRS formulation of SSCWLP in Variation 1)**

Problem Description	% Improvement
5X5X5 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V1.2B	10.21%
5X5X5 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V1.2B	10.24%
10X10X10 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V1.2B	24.12%
10X10X10 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V1.2B	24.18%
20X20X20 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V1.2B	38.80%
20X20X20 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V1.2B	38.84%

From the data obtained above in the analyses phase we can infer the following from the tables 5.1, 5.2, 5.3:

1. for  $\Sigma cap_i = 2.0$  the bounds obtained in Variation 1 and both the Styles for formulations .2B, .4B, .6B are significantly better than those obtained by other

relaxations. *Big-M constraint really boosts the bounds in Variation 1*  
*( $cap_j y_j \geq \sum_{ik} x_{ijk}$ ) for case  $\Sigma cap_j = 2.0$ .*

2. The improvement increases with the problem size so we can hope for more significant improvements for bigger problem sizes which tend to the real world problems.
3. The bounds obtained by in Variation 2 and both the Styles for formulations .2B, .4B, and .6B are worse than bounds obtained by formulations .1B, .3B, and .5B. Hence we see that the strength of  $y_j$  in the capacity constraint for both the Styles and all the formulations is significant and so in Variation 2 of all the formulations we get very poor results for formulations .2B, .4B, and .6B. *Big-M is worse than SRS & WRS in Variation 2 ( $cap_j \geq \sum_{ik} x_{ijk}$ ) for  $\Sigma cap_j = 2.0$ .*

#### *Conclusion 1:*

For both the Styles and both Variations we get same bounds for .2B, .4B and .6B.

#### *Conclusion 2:*

For both the styles and Variation 1 with low capacities ( $\Sigma cap_j = 2.0$ ) and all problem sizes

Bound (.2B, .4B, .6B) are significantly greater than Bound (.1B, .3B, .5B)

### **5.2.2 Statistical Significance of Big M Formulations (Case $\Sigma cap_j = 6.0$ )**

In this section the results for the T-test carried for case  $\Sigma cap_j = 6.0$  between the .1B formulations and .2B for both the Styles and Variations yield the results is shown in Table 5.4

TABLE 5.4(T-test between .1B's and .2B's)

Problem description	T calculated
5X5X5 S1V1.1B AND S1V1.2B	24.23
5X5X5 S2V1.1B AND S2V1.2B	24.22
5X5X5 S1V2.1B AND S1V2.2B	64.46
5X5X5 S2V2.1B AND S2V2.2B	64.90
10X10X10 S1V1.1B AND S1V1.2B	17.70
10X10X10 S2V1.1B AND S2V1.2B	17.76
10X10X10 S1V2.1B AND S1V2.2B	89.81
10X10X10 S2V2.1B AND S2V2.2B	89.78
20X20X20 S1V1.1B AND S1V1.2B	0.47
20X20X20 S2V1.1B AND S2V1.2B	0.26
20X20X20 S1V2.1B AND S1V2.2B	141.79
20X20X20 S2V2.1B AND S2V2.2B	141.56

From the above table we can enlist the above inferences:

1. For  $\Sigma cap_j = 6.0$  the bounds obtained by in Variation 2 and both the Styles for formulations .2B, .4B, and .6B are worse than bounds obtained by formulations .1B, .3B, and .5B. *This implies that Big-M is ineffective in boosting bounds in  $\Sigma cap_j = 6.0$ .*
2. For  $\Sigma cap_j = 6.0$  the bounds obtained in Variation 1 and both the styles for formulations .2B, .4B, .6B are worse than bounds obtained by formulations .1B, .3B, .5B in case when the problem size is 5X5X5 and 10X10X10. But in problem size 20X20X20 there is no significant difference between bounds obtained by the two formulations. We can say that with  $\Sigma cap_j = 6.0$  the problems for small problem sizes can be treated as Uncapacitated problem where any one warehouse location is sufficient to fulfill all the demands. Hence the bounds obtained by the formulations .1B, .3B, .5B perform better



as  $y_j$  takes value 1 for the LP relaxation. *Big-M fails to boost the bounds that is contradictory to the case of what happened in  $\Sigma\text{cap}_j = 2.0$ .*

Hence with the help of the data in chapter 4 and above tables we can give following conclusions

*Conclusion 3:*

For both the styles and both variations we get same bounds for .2B, .4B and .6B.

*Conclusion 4:*

For both the styles and variation 1 with  $\Sigma\text{cap}_j = 6.0$  (high capacities)

Bounds (.1B, .3B, .5B) are significantly greater than Bounds (.2B, .4B, .6B) for small problem sizes (5X5X5 and 10X10X10)

And

Bounds (.1B, .3B, .5B) no significant difference Bounds (.2B, .4B, .6B) for larger problem sizes (20X20X20)

### **5.3 Statistically insignificant difference of Bounds for .1B, .3B and .5B**

This section shows the analysis of the bounds obtained by the weak and the two strong relaxations as they are referred in chapter three. Though there is some improvement in the bounds given by .3B and .5B over the bound given by .1B for both the Styles and Variations and both capacity levels of  $\Sigma\text{cap}_j = 2.0$  and  $\Sigma\text{cap}_j = 6.0$ . But when an F- test is done on the data it is seen that the difference is statistically insignificant. The results of the F-test done is shown as below in Table 5.5

Table 5.5(F-Test between .1B, .3B and .5B)

Problem Description	F-calculated
5X5X5 $\Sigma cap_i = 2.0$ S1V1.1B,.3B,.5B	8.63E-11
5X5X5 $\Sigma cap_i = 2.0$ S2V1.1B,.3B,.5B	0.000113
5X5X5 $\Sigma cap_i = 2.0$ S1V2.1B,.3B,.5B	0.006605
5X5X5 $\Sigma cap_i = 2.0$ S2V2.1B,.3B,.5B	0.00855
5X5X5 $\Sigma cap_i = 6.0$ S1V1.1B,.3B,.5B	0.000532
5X5X5 $\Sigma cap_i = 6.0$ S2V1.1B,.3B,.5B	0.000448
5X5X5 $\Sigma cap_i = 6.0$ S1V2.1B,.3B,.5B	9.24E-05
5X5X5 $\Sigma cap_i = 6.0$ S2V2.1B,.3B,.5B	0.170664
10X10X10 $\Sigma cap_i = 2.0$ S1V1.1B,.3B,.5B	-4.70E-13
10X10X10 $\Sigma cap_i = 2.0$ S2V1.1B,.3B,.5B	3.59E-05
10X10X10 $\Sigma cap_i = 2.0$ S1V2.1B,.3B,.5B	0.012838
10X10X10 $\Sigma cap_i = 2.0$ S2V2.1B,.3B,.5B	0.016373
10X10X10 $\Sigma cap_i = 6.0$ S1V1.1B,.3B,.5B	8.20E-08
10X10X10 $\Sigma cap_i = 6.0$ S2V1.1B,.3B,.5B	0.000782
10X10X10 $\Sigma cap_i = 6.0$ S1V2.1B,.3B,.5B	0.00335
10X10X10 $\Sigma cap_i = 6.0$ S2V2.1B,.3B,.5B	0.003338
20X20X20 $\Sigma cap_i = 2.0$ S1V1.1B,.3B,.5B	2.80E-05
20X20X20 $\Sigma cap_i = 2.0$ S2V1.1B,.3B,.5B	7.40E-06
20X20X20 $\Sigma cap_i = 2.0$ S1V2.1B,.3B,.5B	0.04989
20X20X20 $\Sigma cap_i = 2.0$ S2V2.1B,.3B,.5B	0.052704
20X20X20 $\Sigma cap_i = 6.0$ S1V1.1B,.3B,.5B	2.56E-07
20X20X20 $\Sigma cap_i = 6.0$ S2V1.1B,.3B,.5B	0.000484
20X20X20 $\Sigma cap_i = 6.0$ S1V2.1B,.3B,.5B	0.017559
20X20X20 $\Sigma cap_i = 6.0$ S2V2.1B,.3B,.5B	0.046795

The above values for F-calculated can be compared with the following values of F-critical at the different significance levels (for set of 90 data).

**Table 5.6 (F-critical values)**

Significance level	Alpha= 0.05	Alpha= 0.01	Alpha= 0.005
F-critical	3.029598	4.685546	5.4049

For the data given in tables 5.5 and 5.6 we can infer that there is no significant difference between the bounds obtained by the formulations .1B, .3B and .5B for both the styles and both the variations.

Hence with the help of the data in chapter 4 and above tables we can give following conclusions

*Conclusion 5:*

There is no significant difference between Bound (.1B), Bound (.3B) and Bound (.5B) for both Styles and both the Variations.

#### **5.4 Better Bounds by Variation 1 than Variation 2**

This section shows the analysis of the bounds obtained by the all the relaxations of Variation 1 and Variation 2 and both capacity levels of  $\Sigma \text{cap}_j = 2.0$  and  $\Sigma \text{cap}_j = 6.0$ . It is seen from the analysis that bounds obtained by Variation 1 is significantly better than bounds obtained by relaxations in Variation 2 for both the Styles. This can be attributed to the fact that the addition of  $y_j$  in capacity constraint gives a stronger relaxation for all formulations than the formulations without  $y_j$  in capacity constraint. A T-test was done on data set one by one for the corresponding formulation in Variation 1 and Variation 2 for both the capacity levels. Tables 5.7, 5.8, 5.9 given below establish the above fact.

**Table 5.7(T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 5X5X5)**

Problem Description	T-Calculated
5X5X5 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V2.1B	18.31
5X5X5 $\Sigma cap_i = 2.0$ S1V1.2B AND S1V2.2B	50.83
5X5X5 $\Sigma cap_i = 2.0$ S1V1.3B AND S1V2.3B	18.27
5X5X5 $\Sigma cap_i = 2.0$ S1V1.4B AND S1V2.4B	50.83
5X5X5 $\Sigma cap_i = 2.0$ S1V1.5B AND S1V2.5B	18.27
5X5X5 $\Sigma cap_i = 2.0$ S1V1.6B AND S1V2.6B	50.83
5X5X5 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V2.1B	18.25
5X5X5 $\Sigma cap_i = 2.0$ S2V1.2B AND S2V2.2B	50.83
5X5X5 $\Sigma cap_i = 2.0$ S2V1.3B AND S2V2.3B	18.21
5X5X5 $\Sigma cap_i = 2.0$ S2V1.4B AND S2V2.4B	50.83
5X5X5 $\Sigma cap_i = 2.0$ S2V1.5B AND S2V2.5B	18.21
5X5X5 $\Sigma cap_i = 2.0$ S2V1.6B AND S2V2.6B	50.83
5X5X5 $\Sigma cap_i = 6.0$ S1V1.1B AND S1V2.1B	5.50
5X5X5 $\Sigma cap_i = 6.0$ S1V1.2B AND S1V2.2B	41.82
5X5X5 $\Sigma cap_i = 6.0$ S1V1.3B AND S1V2.3B	5.53
5X5X5 $\Sigma cap_i = 6.0$ S1V1.4B AND S1V2.4B	47.90
5X5X5 $\Sigma cap_i = 6.0$ S1V1.5B AND S1V2.5B	5.40
5X5X5 $\Sigma cap_i = 6.0$ S1V1.6B AND S1V2.6B	47.86
5X5X5 $\Sigma cap_i = 6.0$ S2V1.1B AND S2V2.1B	5.40
5X5X5 $\Sigma cap_i = 6.0$ S2V1.2B AND S2V2.2B	41.89
5X5X5 $\Sigma cap_i = 6.0$ S2V1.3B AND S2V2.3B	3.66
5X5X5 $\Sigma cap_i = 6.0$ S2V1.4B AND S2V2.4B	33.04
5X5X5 $\Sigma cap_i = 6.0$ S2V1.5B AND S2V2.5B	5.35
5X5X5 $\Sigma cap_i = 6.0$ S2V1.6B AND S2V2.6B	47.87

**Table 5.8(T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 10X10X10)**

Problem Description	T-Calculated
10X10X10 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V2.1B	26.38
10X10X10 $\Sigma cap_i = 2.0$ S1V1.2B AND S1V2.2B	52.88
10X10X10 $\Sigma cap_i = 2.0$ S1V1.3B AND S1V2.3B	26.34
10X10X10 $\Sigma cap_i = 2.0$ S1V1.4B AND S1V2.4B	52.88
10X10X10 $\Sigma cap_i = 2.0$ S1V1.5B AND S1V2.5B	26.34
10X10X10 $\Sigma cap_i = 2.0$ S1V1.6B AND S1V2.6B	52.88

Problem Description	T-Calculated
10X10X10 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V2.1B	26.37
10X10X10 $\Sigma cap_i = 2.0$ S2V1.2B AND S2V2.2B	52.89
10X10X10 $\Sigma cap_i = 2.0$ S2V1.3B AND S2V2.3B	26.34
10X10X10 $\Sigma cap_i = 2.0$ S2V1.4B AND S2V2.4B	52.91
10X10X10 $\Sigma cap_i = 2.0$ S2V1.5B AND S2V2.5B	26.33
10X10X10 $\Sigma cap_i = 2.0$ S2V1.6B AND S2V2.6B	52.88
10X10X10 $\Sigma cap_i = 6.0$ S1V1.1B AND S1V2.1B	9.14
10X10X10 $\Sigma cap_i = 6.0$ S1V1.2B AND S1V2.2B	33.08
10X10X10 $\Sigma cap_i = 6.0$ S1V1.3B AND S1V2.3B	9.10
10X10X10 $\Sigma cap_i = 6.0$ S1V1.4B AND S1V2.4B	33.08
10X10X10 $\Sigma cap_i = 6.0$ S1V1.5B AND S1V2.5B	9.09
10X10X10 $\Sigma cap_i = 6.0$ S1V1.6B AND S1V2.6B	33.08
10X10X10 $\Sigma cap_i = 6.0$ S2V1.1B AND S2V2.1B	9.12
10X10X10 $\Sigma cap_i = 6.0$ S2V1.2B AND S2V2.2B	33.05
10X10X10 $\Sigma cap_i = 6.0$ S2V1.3B AND S2V2.3B	9.08
10X10X10 $\Sigma cap_i = 6.0$ S2V1.4B AND S2V2.4B	33.08
10X10X10 $\Sigma cap_i = 6.0$ S2V1.5B AND S2V2.5B	9.16
10X10X10 $\Sigma cap_i = 6.0$ S2V1.6B AND S2V2.6B	33.05

**Table 5.9(T-Test between Bounds of Variation 1 and Variation 2 for All Relaxations, Both Styles and both Capacity Levels; Problem Size 20X20X20)**

Problem Description	T-Calculated
20X20X20 $\Sigma cap_i = 2.0$ S1V1.1B AND S1V2.1B	35.65
20X20X20 $\Sigma cap_i = 2.0$ S1V1.2B AND S1V2.2B	60.09
20X20X20 $\Sigma cap_i = 2.0$ S1V1.3B AND S1V2.3B	35.60
20X20X20 $\Sigma cap_i = 2.0$ S1V1.4B AND S1V2.4B	60.08
20X20X20 $\Sigma cap_i = 2.0$ S1V1.5B AND S1V2.5B	35.63
20X20X20 $\Sigma cap_i = 2.0$ S1V1.6B AND S1V2.6B	60.05
20X20X20 $\Sigma cap_i = 2.0$ S2V1.1B AND S2V2.1B	35.65
20X20X20 $\Sigma cap_i = 2.0$ S2V1.2B AND S2V2.2B	60.06
20X20X20 $\Sigma cap_i = 2.0$ S2V1.3B AND S2V2.3B	35.62
20X20X20 $\Sigma cap_i = 2.0$ S2V1.4B AND S2V2.4B	60.06
20X20X20 $\Sigma cap_i = 2.0$ S2V1.5B AND S2V2.5B	35.61
20X20X20 $\Sigma cap_i = 2.0$ S2V1.6B AND S2V2.6B	60.13
20X20X20 $\Sigma cap_i = 6.0$ S1V1.1B AND S1V2.1B	12.21
20X20X20 $\Sigma cap_i = 6.0$ S1V1.2B AND S1V2.2B	45.78
20X20X20 $\Sigma cap_i = 6.0$ S1V1.3B AND S1V2.3B	12.15

Problem Description	T-Calculated
20X20X20 $\Sigma cap_i = 6.0$ S1V1.4B AND S1V2.4B	45.78
20X20X20 $\Sigma cap_i = 6.0$ S1V1.5B AND S1V2.5B	12.28
20X20X20 $\Sigma cap_i = 6.0$ S1V1.6B AND S1V2.6B	45.78
20X20X20 $\Sigma cap_i = 6.0$ S2V1.1B AND S2V2.1B	12.35
20X20X20 $\Sigma cap_i = 6.0$ S2V1.2B AND S2V2.2B	46.29
20X20X20 $\Sigma cap_i = 6.0$ S2V1.3B AND S2V2.3B	12.27
20X20X20 $\Sigma cap_i = 6.0$ S2V1.4B AND S2V2.4B	46.29
20X20X20 $\Sigma cap_i = 6.0$ S2V1.5B AND S2V2.5B	12.28
20X20X20 $\Sigma cap_i = 6.0$ S2V1.6B AND S2V2.6B	46.29

When we compare the calculated values for the T-statistic and the T-critical given in table 5.2 at the different significance levels (for set of 90 data) we can very well say that the bounds obtained by different formulations for Variation 1 is statistically significant than the bounds obtained by different formulation for Variation 2 for both the Styles and both capacity levels.

Hence with the help of the data in chapter 4 and above tables we can give following conclusion.

*Conclusion 6:*

Bounds (S1V1.1B, .2B, .3B, .4B, .5B, .6B)

Are significantly greater than

Bounds (S1V2.1B, .2B, .3B, .4B, .5B, .6B)

And

Bounds (S2V1.1B, .2B, .3B, .4B, .5B, .6B)

Are significantly greater than

Bounds (S2V2.1B, .2B, .3B, .4B, .5B, .6B)

With corresponding comparison of different formulations.

## 5.5 Comparison of Number of Iterations for Both Styles

This section shows the analysis of the number of iterations needed by the two Styles for all the relaxations of Variation 1 and Variation 2 and both capacity levels. It is seen from the analysis that the number of iterations required for a formulation in Style 2 is often less than number of iterations required for the corresponding formulation in Style 1. To establish this fact a T-test was done on the corresponding formulations for both the Variations and capacity levels. This can be attributed to the more complex nature of Style1 and also to more no of variables and constraints compared to Style 2. The Tables 5.10, 5.11, 5.12 given below illustrate the above fact.

**Table 5.10(T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 5X5X5)**

Problem Description	T-Calculated
5X5X5 $\Sigma cap_i = 2.0$ S1V1.1A AND S2V1.1A	-3.04
5X5X5 $\Sigma cap_i = 2.0$ S1V1.1B AND S2V1.1B	4.01
5X5X5 $\Sigma cap_i = 2.0$ S1V1.2B AND S2V1.2B	-0.85
5X5X5 $\Sigma cap_i = 2.0$ S1V1.3B AND S2V1.3B	6.75
5X5X5 $\Sigma cap_i = 2.0$ S1V1.4B AND S2V1.4B	-5.03
5X5X5 $\Sigma cap_i = 2.0$ S1V1.5B AND S2V1.5B	5.94
5X5X5 $\Sigma cap_i = 2.0$ S1V1.6B AND S2V1.6B	-4.91
5X5X5 $\Sigma cap_i = 2.0$ S1V1.7B AND S2V1.7B	7.65
5X5X5 $\Sigma cap_i = 2.0$ S1V2.1A AND S2V2.1A	-0.74
5X5X5 $\Sigma cap_i = 2.0$ S1V2.1B AND S2V2.1B	6.27
5X5X5 $\Sigma cap_i = 2.0$ S1V2.2B AND S2V2.2B	4.67
5X5X5 $\Sigma cap_i = 2.0$ S1V2.3B AND S2V2.3B	8.41
5X5X5 $\Sigma cap_i = 2.0$ S1V2.4B AND S2V2.4B	5.89
5X5X5 $\Sigma cap_i = 2.0$ S1V2.5B AND S2V2.5B	7.62
5X5X5 $\Sigma cap_i = 2.0$ S1V2.6B AND S2V2.6B	6.91
5X5X5 $\Sigma cap_i = 6.0$ S1V1.1A AND S2V1.1A	-5.15
5X5X5 $\Sigma cap_i = 6.0$ S1V1.1B AND S2V1.1B	3.68
5X5X5 $\Sigma cap_i = 6.0$ S1V1.2B AND S2V1.2B	2.51
5X5X5 $\Sigma cap_i = 6.0$ S1V1.3B AND S2V1.3B	9.07
5X5X5 $\Sigma cap_i = 6.0$ S1V1.4B AND S2V1.4B	-5.48
5X5X5 $\Sigma cap_i = 6.0$ S1V1.5B AND S2V1.5B	5.97
5X5X5 $\Sigma cap_i = 6.0$ S1V1.6B AND S2V1.6B	-8.56
5X5X5 $\Sigma cap_i = 6.0$ S1V1.7B AND S2V1.7B	3.89

Problem Description	T-Calculated
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.1A AND S2V2.1A	3.10
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.1B AND S2V2.1B	4.95
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.2B AND S2V2.2B	-0.20
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.3B AND S2V2.3B	9.06
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.4B AND S2V2.4B	10.53
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.5B AND S2V2.5B	8.62
5X5X5 $\Sigma_{cap_i}=6.0$ S1V2.6B AND S2V2.6B	4.70

**Table 5.11(T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 10X10X10)**

Problem Description	T-Calculated
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.1A AND S2V1.1A	10.21
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.1B AND S2V1.1B	16.57
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.2B AND S2V1.2B	13.93
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.3B AND S2V1.3B	19.18
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.4B AND S2V1.4B	-6.80
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.5B AND S2V1.5B	8.16
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.6B AND S2V1.6B	-10.21
10X10X10 $\Sigma_{cap_i}=2.0$ S1V1.7B AND S2V1.7B	13.65
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.1A AND S2V2.1A	4.35
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.1B AND S2V2.1B	12.23
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.2B AND S2V2.2B	6.46
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.3B AND S2V2.3B	11.56
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.4B AND S2V2.4B	13.26
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.5B AND S2V2.5B	9.69
10X10X10 $\Sigma_{cap_i}=2.0$ S1V2.6B AND S2V2.6B	10.82
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.1A AND S2V1.1A	13.62
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.1B AND S2V1.1B	12.85
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.2B AND S2V1.2B	10.51
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.3B AND S2V1.3B	13.36
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.4B AND S2V1.4B	-4.94
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.5B AND S2V1.5B	8.08
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.6B AND S2V1.6B	-7.81
10X10X10 $\Sigma_{cap_i}=6.0$ S1V1.7B AND S2V1.7B	10.80
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.1A AND S2V2.1A	11.36
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.1B AND S2V2.1B	12.29
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.2B AND S2V2.2B	8.18
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.3B AND S2V2.3B	12.54
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.4B AND S2V2.4B	16.83
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.5B AND S2V2.5B	8.33
10X10X10 $\Sigma_{cap_i}=6.0$ S1V2.6B AND S2V2.6B	16.87



**Table 5.12(T-Test on number of iterations for All corresponding Relaxations of Style1 and Style 2 and both Capacity Levels; Problem Size 20X20X20)**

Problem Description	T-Calculated
20X20X20 $\Sigma cap_i = 2.0$ S1V1.1A AND S2V1.1A	13.52
20X20X20 $\Sigma cap_i = 2.0$ S1V1.1B AND S2V1.1B	11.91
20X20X20 $\Sigma cap_i = 2.0$ S1V1.2B AND S2V1.2B	18.79
20X20X20 $\Sigma cap_i = 2.0$ S1V1.3B AND S2V1.3B	14.36
20X20X20 $\Sigma cap_i = 2.0$ S1V1.4B AND S2V1.4B	3.59
20X20X20 $\Sigma cap_i = 2.0$ S1V1.5B AND S2V1.5B	14.59
20X20X20 $\Sigma cap_i = 2.0$ S1V1.6B AND S2V1.6B	4.41
20X20X20 $\Sigma cap_i = 2.0$ S1V1.7B AND S2V1.7B	8.80
20X20X20 $\Sigma cap_i = 2.0$ S1V2.1A AND S2V2.1A	2.71
20X20X20 $\Sigma cap_i = 2.0$ S1V2.1B AND S2V2.1B	11.75
20X20X20 $\Sigma cap_i = 2.0$ S1V2.2B AND S2V2.2B	20.16
20X20X20 $\Sigma cap_i = 2.0$ S1V2.3B AND S2V2.3B	13.21
20X20X20 $\Sigma cap_i = 2.0$ S1V2.4B AND S2V2.4B	8.71
20X20X20 $\Sigma cap_i = 2.0$ S1V2.5B AND S2V2.5B	12.44
20X20X20 $\Sigma cap_i = 2.0$ S1V2.6B AND S2V2.6B	13.35
20X20X20 $\Sigma cap_i = 6.0$ S1V1.1A AND S2V1.1A	12.23
20X20X20 $\Sigma cap_i = 6.0$ S1V1.1B AND S2V1.1B	10.19
20X20X20 $\Sigma cap_i = 6.0$ S1V1.2B AND S2V1.2B	18.77
20X20X20 $\Sigma cap_i = 6.0$ S1V1.3B AND S2V1.3B	16.65
20X20X20 $\Sigma cap_i = 6.0$ S1V1.4B AND S2V1.4B	2.94
20X20X20 $\Sigma cap_i = 6.0$ S1V1.5B AND S2V1.5B	7.49
20X20X20 $\Sigma cap_i = 6.0$ S1V1.6B AND S2V1.6B	2.39
20X20X20 $\Sigma cap_i = 6.0$ S1V1.7B AND S2V1.7B	7.16
20X20X20 $\Sigma cap_i = 6.0$ S1V2.1A AND S2V2.1A	7.17
20X20X20 $\Sigma cap_i = 6.0$ S1V2.1B AND S2V2.1B	6.99
20X20X20 $\Sigma cap_i = 6.0$ S1V2.2B AND S2V2.2B	16.95
20X20X20 $\Sigma cap_i = 6.0$ S1V2.3B AND S2V2.3B	13.54
20X20X20 $\Sigma cap_i = 6.0$ S1V2.4B AND S2V2.4B	10.00
20X20X20 $\Sigma cap_i = 6.0$ S1V2.5B AND S2V2.5B	11.00
20X20X20 $\Sigma cap_i = 6.0$ S1V2.6B AND S2V2.6B	14.81

The above values for T-calculated can be compared with the following values of T-critical at the different significance levels (for set of 90 data) given in Table 5.2.

From the data obtained above in the analyses phase we can infer the following from the tables 5.10, 5.11, 5.12:

1. For problem Size 20X20X20 for all formulations and both the variations the number of iterations for Style 1 is significantly higher than for Style 2.
2. For problem size 5X5X5 and 10X10X10 there are some instances when the number of iterations for Style 2 is greater than Style1, but in most formulations number of iterations for Style 1 is greater than Style2. The first case is encountered in formulations .4B and .6B on some instances.
3. For higher problem size the number of instances where number of iterations for Style 2 is greater than Style1 is very low or almost absent and so it can be said that for higher problem sizes number of iterations for Style 1 is greater than Style2 for all the formulations.

Hence with the help of the data in chapter 4 and above tables we can give following conclusion.

*Conclusion 7:*

Numbers of Iterations (Style 1)  
Are significantly greater than  
Number of Iterations (Style 2)

We found that all seven conclusions are fully supported by the statistical tools.

## Chapter 6

### NEW FORMULATION

Encouraged by the results obtained in chapter four for Big M formulations in this chapter we have tried a new formulation only for Style 1 and Variation 2 so as to see the effect of

the bounds given when the Big M formulations are combined with the strong relaxation.

So the new formulation can be listed as below:

#### FORMULATION NEW

So when we replace the constraint (6) and replace it by (10) and (11) as given below the problem becomes

MINIMIZE (0)

Subject to (1) to (4), (8), (10), (11) and (14)

This was done to see if there is some further improvement in the bounds given by Formulation S1V2.3B after adding the Big M constraint with the constraint 10.

For the analyses fifty problems for problem size 20X20X20 that are randomly generated are tried and results are given in Appendix 2.

On the analyses of the result we see that although there is no improvement in the bounds given by the new formulation over the bounds given by S1V2.1B, S1V2.3B and S1V2.5B, but the number of iterations needed for the new formulation is significantly less than the number of iterations needed for the formulation S1V2.3B. Since in chapter 5 we have already concluded that there is no significant difference between the bounds

given by .1B, .3B, .5B; we have done a T-test of the bounds given by .3B with bounds given by new formulation only and assume it to be applicable to .1B and .5B also.

Thus we establish that there is no significant difference in the bounds given by the two formulations.

The following table shows the results of the T-test done on the new formulation and S1V2.3B for both the objective value and the number of iterations.

**Table 6.1(T-test for the New Formulation)**

OBJ VALUE	T- Calculated
20X20X20 CAP2 S1V2.3B AND NEW	-0.16685
NO OF ITERATIONS	
20X20X20 CAP2 S1V2.3B AND NEW	3.227632

If we compare the above T- calculated with the T-critical values at different significance levels we see that the Bounds given by the two formulations are not statistically different but the number of iterations needed by the new formulation is significantly less than the number of iterations needed by S1V2.3B.

The T- critical values at different significance levels for a data size of fifty is given as below:

**Table 6.2(T-critical data size 50)**

Significance level	Alpha= 0.05	Alpha= 0.01	Alpha= 0.005
T-critical(one tail)	1.676551	2.404886	2.679953
T-critical(two tailed)	2.009574	2.679953	2.939742

## Chapter 7

### **DIRECTIONS FOR FUTURE RESEARCH**

Following future research topics are suggested:

1. To develop a heuristic to establish bounds given by formulations S1V1.2B, .4B, .6B and S2V1.2B, .4B, .6B (Big M formulations) for low capacities as they give stronger relaxation bounds than the SRS.
2. To give the theoretical proofs to establish the empirical finding in the present work.

## REFERENCES

1. Geoffrion, A.M. and Graves G.W. "Multicommodity distribution system design by Benders Decomposition". *Math Programming*, 2(1974).82-114.
2. Sharma R.R.K., "Modelling a fertiliser distribution system". *European Journal of Operation Research*, (1991).
3. Christofides, N. and Beasley, J.E., "Extensions to a Lagrangean relaxation approach for the capacitated warehouse location problem", *European Journal of Operation Research*, 12, 1(1983), 19-28.
4. Sharma R.R.K., "Distribution of food grains in India: An Operational Study", (1996)
5. Hidaka Kazuyoshi and Hiroyuki Okano, "Simulation-based approach to the warehouse location problem for a large-scale real instance". *Proceedings of the 1997 Winter Simulation Conference*.(1997)
6. Kratica Jozef, "Solving the Uncapacitated warehouse location problem by SGA with add-heuristic".(1998)
7. Chhajer, B., Francis, G.I. and Lowe, A.C. "Contributions of operations research to location analysis", *Location Science*, 1(1993), 263-287.
8. Cornuejols, G, Nemhauser, G.L. and Wolsey, L.A. "The Uncapacitated facility location problem", MSSR 493, Graduate School of Industrial Administration, CMU (1983).
9. Cornuejols, G. Fisher, M.L. and Nemhauser, G.L. "Location of Bank accounts to optimise float: An analytic study of exact and approximate algorithms", *Management Science*, 23(1977), 789-810.
10. Feldman, E, Leher, F.A. and Ray T.L. "Warehouse location under continuous economies of scale", *Management Science*, 2(1966), 670-684.
11. Khumawallah, B.M. "An efficient heuristic procedure for capacitated warehouse location problem", *Naval Research Logistics Quarterly*, 21(4) (1974), 609-623.
12. Krarup, J. and Pruzan, P.M. "The simple plant location problem: Survey and synthesis", *European Journal of Operations Research*, 12(1983), 36-81.
13. Kuehn, A.A and Hamburger, M.J. "A heuristic program for locating warehouses", *Management Science*, 9(1963), 643-666.
14. Erlenkotter, D. "A dual based procedure for uncapacitated facility location", *Operations Research*, 26(1978), 992-1009.

# **APPENDIX 1**

## STYLE 1, VARIATION 1(5X5X5, $\Sigma_{cap}=2.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	15483	130	12211	37	12499	51	12211	165	12499	35	12211	181	12499	44	12211	35
2	15689	129	10530	37	10792	28	10530	81	10792	127	10530	91	10792	50	10530	64
3	19900	146	14413	23	14917	35	14413	199	14917	35	14413	192	14917	45	14413	17
4	17438	122	14242	46	14427	81	14242	88	14427	60	14242	219	14427	55	14242	23
5	12810	106	7443	32	8776	32	7443	122	8776	38	7443	259	8776	89	7443	40
6	16461	166	13562	30	14402	55	13562	79	14402	124	13562	144	14402	73	13562	31
7	12235	122	6722	26	7301	33	6722	61	7301	32	6722	130	7301	40	6722	39
8	10649	99	8106	27	9069	91	8106	83	9069	33	8106	108	9069	48	8106	76
9	12810	106	7443	32	8776	32	7443	122	8776	38	7443	259	8776	89	7443	40
10	15881	100	11732	27	11884	53	11732	156	11884	32	11732	144	11884	38	11732	25
<b><math>\Sigma_{si}=1.5</math></b>																
1	13758	96	9757	20	12572	67	9757	74	12572	36	9757	158	12572	50	9757	35
2	13670	60	10862	38	11910	39	10862	69	11910	63	10862	114	11910	35	10862	22
3	14829	116	13630	18	13984	53	13630	59	13984	78	13630	53	13984	35	13630	24
4	14127	83	10670	35	10948	51	10670	164	10948	71	10670	70	10948	45	10670	25
5	14089	91	11898	25	12820	54	11898	68	12820	44	11898	170	12820	50	11898	27
6	18206	99	13117	24	13773	74	13117	147	13773	68	13117	132	13773	42	13117	22
7	13670	60	10862	38	11910	39	10862	69	11910	63	10862	114	11910	35	10862	22
8	17058	76	15003	33	15962	75	15003	106	15962	64	15003	66	15962	51	15003	29
9	12897	102	9878	36	11027	71	9878	104	11027	48	9878	107	11027	65	9878	51
10	10649	99	8106	27	9069	91	8106	83	9069	33	8106	108	9069	48	8106	76
<b><math>\Sigma_{si}=2.0</math></b>																
1	13737	107	7064	34	7477	19	7064	70	7477	35	7064	98	7477	32	7064	23
2	13544	80	11162	14	11631	27	11162	101	11631	24	11162	62	11631	38	11162	20
3	13675	83	8932	20	10605	33	8932	85	10605	29	8932	86	10605	26	8932	22
4	13875	90	8249	24	10158	33	8249	156	10158	34	8249	139	10158	40	8249	22
5	15372	94	12361	51	12662	60	12361	153	12662	44	12361	125	12662	41	12361	67
6	12949	68	8501	32	8638	23	8501	102	8638	21	8501	61	8638	43	8501	17
7	13675	83	8932	20	10605	33	8932	85	10605	29	8932	86	10605	26	8932	22
8	16752	78	15648	22	15680	68	15648	114	15680	67	15648	143	15680	39	15648	19
9	14780	105	9913	28	11537	38	9913	145	11537	39	9913	165	11537	50	9913	15
10	16727	75	10327	29	10795	37	10327	149	10795	36	10327	54	10795	49	10327	25
<b><math>\Sigma_{si}=5.0</math></b>																
1	13025	72	7121	27	8452	19	7121	135	8452	37	7121	108	8452	29	7121	23
2	13646	59	10539	15	11401	26	10539	46	11401	32	10539	69	11401	27	10539	13
3	12652	69	7711	19	8479	23	7711	84	8479	41	7711	91	8479	38	7711	19
4	11849	65	9703	4	10144	43	9703	88	10144	25	9703	98	10144	23	9703	25
5	13025	72	7121	27	8452	19	7121	135	8452	37	7121	108	8452	29	7121	23
6	14203	78	8709	32	8976	20	8709	71	8976	25	8709	63	8976	30	8709	14
7	11319	73	6176	29	7858	22	6176	70	7858	24	6176	110	7858	33	6176	17
8	10906	76	6203	30	6568	45	6203	93	6568	20	6203	122	6568	19	6203	22
9	13353	92	10609	21	11222	31	10609	37	11222	43	10609	70	11222	28	10609	18
10	17015	81	11718	33	13466	48	11718	81	13466	28	11718	78	13466	24	11718	19



**$\Sigma si=10.0$** 

1	13162	67	8603	22	8784	19	8603	59	8784	49	8603	79	8784	24	8603	28
2	12850	66	8126	16	10557	21	8126	61	10557	35	8126	46	10557	26	8126	22
3	12850	66	8126	16	10557	21	8126	61	10557	35	8126	46	10557	26	8126	22
4	11160	41	10002	14	11039	32	10002	50	11039	35	10002	92	11039	37	10002	18
5	13531	68	10208	21	10875	20	10208	54	10875	54	10208	46	10875	25	10208	31
6	14161	70	8599	20	9295	36	8599	55	9295	30	8599	66	9295	29	8599	28
7	14152	58	8596	16	9299	45	8596	57	9299	29	8596	114	9299	27	8596	23
8	13522	45	11670	20	12663	19	11670	43	12663	42	11670	130	12663	22	11670	42
9	13514	40	11559	19	13514	32	11559	72	13514	29	11559	89	13514	33	11559	29
10	11169	56	6314	14	6701	21	6314	38	6701	36	6314	94	6701	24	6314	21

 **$\Sigma si=20.0$** 

1	13522	46	10600	18	11522	22	10600	57	11522	55	10600	39	11522	38	10600	27
2	13514	62	7715	19	9683	22	7715	44	9683	38	7715	72	9683	20	7715	29
3	14516	71	8124	27	8638	30	8124	44	8638	38	8124	88	8638	40	8124	52
4	14518	68	8131	27	8643	33	8131	46	8643	44	8131	57	8643	33	8131	50
5	12850	61	7722	16	10064	34	7722	77	10064	39	7722	76	10064	30	7722	22
6	12848	55	7721	16	10063	34	7721	56	10063	39	7721	55	10063	36	7721	22
7	13520	77	9254	16	10193	36	9254	90	10193	28	9254	66	10193	29	9254	19
8	13520	77	9254	16	10193	36	9254	90	10193	28	9254	66	10193	29	9254	19
9	15839	76	9973	22	10432	56	9973	101	10432	21	9973	99	10432	30	9973	25
10	12499	75	9403	16	10818	70	9403	44	10818	47	9403	49	10818	34	9403	28

 **$\Sigma si=30.0$** 

1	13136	64	9370	19	9788	47	9370	58	9788	18	9370	48	9788	26	9370	23
2	13152	60	8788	19	9084	41	8788	53	9084	42	8788	46	9084	32	8788	24
3	14157	65	11083	21	11753	23	11083	91	11753	23	11083	48	11753	22	11083	32
4	13143	69	9725	18	10295	49	9725	105	10295	47	9725	47	10295	45	9725	23
5	13143	69	9725	18	10295	49	9725	105	10295	47	9725	47	10295	45	9725	23
6	13144	55	10741	19	11614	23	10741	46	11614	19	10741	90	11614	32	10741	29
7	13518	48	11186	18	11500	19	11186	89	11500	37	11186	86	11500	43	11186	23
8	13157	53	8977	19	9843	19	8977	48	9843	38	8977	51	9842	19	8977	42
9	18055	132	9206	14	11351	26	9206	104	11351	45	9206	55	11351	29	9206	18
10	15032	108	9365	16	11250	30	9365	52	11250	32	9365	78	11250	28	9365	20

 **$\Sigma si=40.0$** 

1	14156	63	9855	16	10464	28	9855	53	10464	31	9855	67	10464	32	9855	18
2	11164	42	9680	14	11037	35	9680	63	11037	43	9680	82	11037	25	9680	28
3	11166	50	8835	14	9359	29	8835	54	9359	20	8835	44	9359	34	8835	28
4	11166	50	8835	14	9359	29	8835	54	9359	20	8835	44	9359	34	8835	27
5	14521	63	10551	16	11622	24	10551	59	11622	64	10551	43	11622	24	10551	35
6	13527	60	9177	14	10114	19	9177	94	10114	35	9177	95	10114	19	9177	30
7	13528	62	9181	19	10119	20	9181	44	10119	21	9181	56	10119	31	9181	22
8	13149	62	8973	16	9179	34	8973	51	9179	24	8973	56	9179	40	8973	32
9	13526	54	8974	28	9383	18	8974	42	9383	57	8974	96	9383	23	8974	26
10	13146	69	9278	19	9712	48	9278	46	9712	42	9278	75	9712	28	9278	36

 **$\Sigma si=50.0$** 

1	17174	72	10595	31	11940	32	10595	37	11940	61	10595	44	11940	57	10595	21
2	18492	61	10107	33	10422	26	10107	25	10422	54	10107	35	10422	67	10107	25
3	13189	61	8723	31	9108	31	8723	25	9108	64	8723	68	9108	55	8723	23
4	11395	62	8860	29	9782	24	8860	65	9782	49	8860	39	9782	49	8860	19
5	17028	64	12641	13	13893	34	12641	28	13893	55	12641	48	13893	66	12641	20
6	11765	45	8435	22	9323	29	8435	50	9323	47	8435	48	9323	78	8435	19
7	13858	48	10041	22	12118	36	10041	52	12118	49	10041	62	12118	69	10041	26
8	12293	45	7600	20	8064	34	7600	51	8064	34	7600	55	8064	63	7600	10
9	15352	47	11735	17	12994	47	11735	61	12994	42	11735	94	12994	55	11735	9
10	15356	58	11754	19	12998	45	11754	51	12998	58	11754	89	12998	57	11754	14

## STYLE 2, VARIATION 1(5X5X5, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	15469	104	12186	13	12485	61	12186	61	12485	35	12282	87	12485	36	12186	13
2	15664	78	10500	23	10769	52	10500	29	10770	58	10527	43	10770	46	10500	14
3	19891	66	14401	18	14908	63	14401	81	14908	72	14403	40	14908	32	14401	16
4	17424	61	14225	20	14418	37	14225	44	14418	44	14236	58	14418	48	14225	17
5	12796	85	7419	21	8761	38	7419	49	8761	44	7439	41	8761	54	7419	13
6	16454	101	13543	18	14400	33	13543	83	14400	69	13546	68	14400	58	13543	9
7	12228	110	6713	26	7292	26	6713	57	7292	92	6720	42	7292	39	6713	9
8	10634	80	8080	10	9053	19	8080	65	9053	69	8097	38	9053	34	8080	12
9	12796	85	7419	21	8761	38	7419	49	8761	44	7439	41	8761	54	7419	13
10	15864	77	11706	20	11862	31	11706	83	11862	81	11726	51	11862	80	11706	15
<b><math>\Sigma_{si}=1.5</math></b>																
1	13756	76	9754	18	12570	38	9754	55	12571	48	9754	45	12570	48	9754	12
2	13658	89	10845	8	11898	47	10845	50	11898	56	10859	46	11898	67	10845	8
3	14821	122	13616	8	13975	48	13616	48	13975	51	13627	61	13975	56	13616	8
4	14125	68	10665	18	10945	29	10665	47	10945	40	10668	62	10945	34	10665	18
5	14083	103	11893	14	12814	31	11893	54	12814	50	11894	60	12814	35	11893	14
6	18204	112	13113	12	13763	38	13113	48	13763	38	13113	40	13763	41	13115	12
7	13658	89	10845	8	11898	47	10845	50	11898	56	10859	46	11898	67	10845	8
8	17055	97	14998	17	15956	42	14998	48	15956	50	15010	48	15956	38	14998	12
9	12892	85	9867	20	11023	45	9867	48	11023	50	9867	48	11023	52	9867	13
10	10634	80	8080	10	9053	19	8080	65	9053	69	8097	38	9053	34	8080	12
<b><math>\Sigma_{si}=2.0</math></b>																
1	13735	81	7061	17	7475	27	7061	93	7475	64	7063	55	7475	43	7061	29
2	13537	99	11154	30	11624	62	11154	61	11624	55	11155	44	11624	28	11154	9
3	13673	66	8930	12	10605	19	8930	101	10605	44	8931	110	10605	57	8930	12
4	13873	74	8246	10	10157	25	8246	70	10157	84	8247	94	10157	86	8246	12
5	15371	72	12358	27	12662	52	12358	69	12662	50	12358	96	12662	62	12358	31
6	12948	61	8488	14	8626	25	8488	83	8626	42	8500	68	8626	60	8488	14
7	13673	66	8930	12	10605	19	8930	101	10605	44	8931	110	10605	57	8930	12
8	16752	84	15648	18	15680	32	15648	42	15680	63	15648	50	15680	32	15648	18
9	14774	66	9819	16	11528	39	9819	49	11528	58	9828	58	11528	37	9819	18
10	16727	62	10326	14	10794	32	10326	67	10794	56	10326	58	10794	37	10326	15
<b><math>\Sigma_{si}=5.0</math></b>																
1	13023	66	7119	13	8450	34	7119	55	8450	36	7121	70	8450	79	7119	14
2	13646	75	10539	10	11401	60	10539	40	11401	56	10539	96	11402	27	10539	8
3	12652	75	7711	16	8479	30	7711	43	8479	61	7711	60	8479	51	7711	14
4	11489	76	9703	29	10144	30	9703	35	10144	25	9703	62	10144	35	9703	37
5	13023	66	7119	13	8450	34	7119	55	8450	36	7121	70	8450	79	7119	14
6	14202	77	8704	35	8972	45	8704	60	8972	46	8708	53	8972	41	8704	38
7	11319	101	6176	13	7858	32	6176	42	7858	75	6176	64	7858	45	6176	14
8	10906	68	6203	34	6568	43	6203	66	6568	36	6203	74	6568	46	6203	33
9	13343	101	10595	14	11211	30	10595	54	11211	63	10608	113	11211	35	10595	14
10	17015	65	11718	18	13466	30	11718	52	13466	36	11718	90	13466	25	11718	11

$\Sigma si=10.0$ 

1	13162	103	8603	33	8784	40	8603	48	8784	36	8603	74	8784	122	8603	23
2	12850	93	8126	14	10557	43	8126	52	10557	65	8126	65	10557	46	8126	11
3	12850	93	8126	14	10557	43	8126	52	10557	65	8126	65	10557	46	8126	9
4	11160	77	10002	14	11039	38	10002	58	11039	49	10002	59	11039	47	10002	11
5	13531	70	10208	14	10875	34	10208	41	10875	107	10208	40	10875	41	10208	12
6	14161	129	8599	14	9295	61	8599	47	9295	56	8599	65	9295	32	8599	10
7	14152	95	8596	14	9299	30	8596	35	9299	58	8596	111	9299	43	8596	13
8	13522	53	11670	33	12663	47	11670	51	12663	56	11670	76	12663	44	11670	25
9	13514	54	11559	14	13514	42	11559	58	13514	45	11559	48	13514	37	11559	12
10	11169	76	6314	14	6701	36	6314	47	6701	56	6314	65	6701	46	6314	14

 $\Sigma si=20.0$ 

1	13522	62	10600	14	11522	34	10600	52	11522	50	10600	57	11522	37	10600	12
2	13514	117	7715	14	9683	43	7715	38	9683	54	7715	43	9683	35	7715	12
3	14516	133	8124	16	8638	35	8124	39	8638	64	8124	36	8638	35	8124	15
4	14518	96	8131	16	8643	34	8131	47	8643	61	8131	43	8643	36	8131	15
5	12850	88	7722	14	10064	38	7722	42	10064	54	7722	49	10064	52	7722	9
6	12848	90	7721	14	10063	38	7721	68	10063	80	7721	52	10063	51	7721	10
7	13520	95	9254	14	10193	36	9254	42	10193	64	9254	46	10193	52	9254	12
8	13520	95	9254	14	10193	36	9254	42	10193	64	9254	46	10193	52	9254	14
9	15839	110	9973	41	10432	38	9973	34	10432	38	9973	29	10432	43	9973	32
10	12499	106	9403	16	10818	34	9403	68	10818	54	9403	36	10818	36	9403	16

 $\Sigma si=30.0$ 

1	13136	84	9370	22	9788	50	9370	44	9788	67	9370	50	9788	48	9370	18
2	13152	66	8788	22	9084	48	8788	45	9084	47	8788	35	9084	59	8788	18
3	14157	107	11083	12	11753	28	11083	60	11753	22	11083	43	11753	53	11083	9
4	13143	68	9725	31	10295	42	9725	39	10295	52	9725	67	10295	39	9725	25
5	13143	68	9725	31	10295	42	9725	39	10295	52	9725	67	10295	39	9725	25
6	13144	60	10741	33	11614	71	10741	56	11614	49	10741	74	11614	50	10741	28
7	13518	62	11186	15	11500	39	11186	71	11500	39	11186	66	11500	43	11186	16
8	13157	71	8977	14	9843	39	8977	75	9843	45	8977	79	9842	45	8977	12
9	18055	158	9206	12	11351	39	9206	43	11351	45	9206	52	11351	29	9206	9
10	15032	125	9365	31	11250	42	9365	44	11250	39	9365	67	11250	43	9365	18

 $\Sigma si=40.0$ 

1	14156	86	9855	15	10464	71	9855	32	10464	39	9855	51	10464	62	9855	17
2	11164	92	9680	16	11037	82	9680	62	11037	43	9680	47	11037	40	9680	21
3	11166	81	8835	14	9359	50	8835	43	9359	57	8835	51	9359	63	8835	12
4	11166	81	8835	14	9359	50	8835	43	9359	57	8835	51	9359	63	8835	12
5	14521	119	10551	14	11622	33	10551	38	11622	59	10551	50	11622	52	10551	12
6	13527	66	9177	33	10114	49	9177	47	10114	76	9177	46	10114	59	9177	23
7	13528	66	9181	33	10119	39	9181	64	10119	45	9182	39	10119	42	9181	23
8	13149	94	8973	12	9179	48	8973	52	9179	84	8973	50	9179	42	8973	12
9	13526	70	8974	15	9383	34	8974	48	9383	51	8974	49	9383	60	8974	9
10	13146	68	9278	39	9712	46	9278	48	9712	44	9278	54	9712	32	9278	24

 $\Sigma si=50.0$ 

1	17174	111	10595	9	11940	20	10595	58	11940	46	10595	92	11940	63	10595	8
2	18492	67	10107	14	10422	28	10107	33	10422	48	10107	43	10422	63	10107	12
3	13189	78	8723	14	9108	23	8723	50	9108	50	8723	57	9108	61	8723	12
4	11395	86	8860	18	9782	27	8860	42	9782	71	8860	51	9782	79	8860	16
5	17028	82	12641	23	13893	39	12641	71	13893	50	12641	52	13893	64	12641	18
6	11765	63	8435	17	9323	37	8435	43	9323	40	8435	46	9323	44	8435	17
7	13858	57	10041	17	12118	50	10041	54	12118	41	10041	68	12118	64	10041	17
8	12293	71	7600	21	8064	34	7600	43	8064	46	7600	58	8064	69	7600	19
9	15352	79	11735	25	12994	36	11735	31	12994	56	11735	59	12994	71	11735	25
10	15356	100	11754	20	12998	38	11754	34	12998	62	11754	47	12998	72	11754	20

## STYLE 1, VARIATION 2(5X5X5, $\Sigma$ cap=2.0)

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma</math>si=1.0</b>														
1	15483	168	7602	38	434	36	7605	61	434	80	7611	134	434	71
2	15689	141	8041	71	423	41	8046	64	423	127	8054	197	423	57
3	19900	261	8705	63	418	44	8719	72	418	35	8725	189	418	62
4	17424	142	8285	96	420	45	8298	65	420	189	8308	241	420	77
5	12810	161	6145	53	425	101	6151	76	425	111	6159	233	425	79
6	16461	265	7042	100	412	57	7050	78	412	85	7057	143	412	91
7	12235	142	6125	88	418	52	6132	75	419	128	6132	197	419	62
8	10649	88	5521	91	420	46	5527	103	420	93	5536	176	420	103
9	12810	161	6145	53	425	101	6151	76	425	111	6159	233	425	79
10	15881	142	8038	32	435	45	8042	53	435	77	8048	97	435	74
<b><math>\Sigma</math>si=1.5</b>														
1	13758	151	6648	39	429	32	6660	62	429	83	6666	67	429	60
2	13670	155	6639	37	414	49	6651	83	414	81	6658	83	414	80
3	14829	157	7348	56	421	37	7363	79	421	66	7370	147	421	55
4	14127	133	6527	42	416	46	6539	73	415	77	6543	79	416	76
5	14089	191	6552	51	417	65	6562	171	417	72	6568	77	417	69
6	18206	188	8655	41	420	41	8667	69	420	84	8675	89	420	76
7	13670	155	6639	37	414	49	6651	83	414	81	6658	83	414	80
8	17058	147	8189	29	413	33	8200	61	413	88	8207	73	413	67
9	12897	135	6609	51	417	51	6616	70	417	82	6621	217	417	76
10	10649	88	5521	91	420	46	5527	103	420	93	5536	176	420	103
<b><math>\Sigma</math>si=2.0</b>														
1	13737	140	6767	33	427	32	6768	94	427	77	6768	75	427	81
2	13544	104	6944	42	431	32	6947	59	431	57	6952	72	431	74
3	13675	109	6676	40	422	31	6680	81	422	85	6683	72	422	85
4	13875	123	6567	29	413	35	6570	60	413	79	6573	51	413	75
5	15372	133	6957	43	418	38	6969	65	418	86	6974	72	418	63
6	12949	115	6657	42	415	26	6668	58	415	176	6680	127	415	79
7	13675	109	6676	40	422	31	6680	81	422	85	6683	72	422	85
8	16752	185	7495	32	417	46	7503	135	417	64	7507	165	417	76
9	14780	119	7618	28	420	45	7619	62	420	64	7625	89	420	89
10	16727	116	6545	38	413	28	6549	82	414	64	6552	72	414	44
<b><math>\Sigma</math>si=5.0</b>														
1	13025	111	6454	21	418	30	6456	65	417	61	6456	56	418	62
2	13646	96	6738	35	413	46	6745	57	413	64	6745	57	413	54
3	12652	85	6936	29	415	21	6938	67	415	60	6939	63	415	66
4	11849	89	6072	64	422	21	6076	43	422	58	6078	50	422	49
5	13025	111	6454	21	418	30	6456	65	417	61	6456	56	418	62
6	14203	101	6975	30	424	25	6983	54	424	66	6985	128	424	72
7	11319	87	5579	22	432	34	5581	56	432	72	5581	49	432	73
8	10906	94	5667	25	420	20	5673	51	420	54	5674	45	420	59
9	13353	108	6859	65	417	54	6871	122	417	50	6873	52	417	55
10	17015	106	8695	46	424	27	8698	106	424	125	8698	112	424	70

$\Sigma si=10.0$ 

1	13162	100	6575	33	337	23	6598	56	337	73	6599	56	337	67
2	12850	85	6215	39	336	23	6226	57	336	74	6226	49	336	51
3	12850	85	6215	39	336	23	6226	57	336	74	6226	49	336	51
4	11160	66	5777	41	335	23	5789	64	335	72	5789	65	335	46
5	13531	94	7071	29	333	23	7085	61	333	72	7085	55	333	70
6	14161	128	6003	49	344	27	6014	61	344	46	6017	128	344	73
7	14152	113	6004	29	342	27	6011	68	342	53	6011	126	342	52
8	13522	86	6654	25	334	29	6673	50	334	60	6673	111	334	50
9	13514	98	6378	24	335	23	6391	49	335	66	6391	99	335	80
10	11169	68	5789	17	344	44	5797	55	344	61	5797	95	344	118

 $\Sigma si=20.0$ 

1	13522	85	6866	34	339	26	6884	45	339	59	6884	95	339	63
2	13514	102	6176	37	338	23	6186	70	338	67	6186	96	338	73
3	14516	101	6294	41	338	24	6307	70	338	67	6307	97	338	57
4	14518	102	6298	44	337	25	6312	45	337	55	6312	94	337	73
5	12850	76	6080	36	336	25	6088	48	336	56	6088	55	336	69
6	12848	77	6078	33	335	24	6087	45	335	67	6087	94	335	57
7	13520	116	6167	25	339	29	6179	64	339	51	6186	119	339	59
8	13520	116	6167	25	339	29	6179	64	339	51	6186	119	339	59
9	15839	142	6686	31	346	28	6705	54	346	65	6705	96	346	48
10	12499	115	6163	29	346	26	6177	44	346	65	6179	136	346	66

 $\Sigma si=30.0$ 

1	13136	113	6132	23	337	20	6144	55	337	50	6144	50	337	60
2	13152	98	6212	45	337	23	6234	52	337	69	6234	96	337	60
3	14157	104	6567	26	338	29	6587	46	338	73	6587	102	338	59
4	13143	81	6259	22	335	45	6283	52	335	66	6283	101	335	53
5	13143	81	6259	22	335	45	6283	52	335	66	6283	101	335	53
6	13144	93	6239	22	334	20	6259	65	334	61	6259	50	334	74
7	13518	87	6410	30	334	23	6431	56	334	61	6431	50	334	71
8	13157	89	7191	32	341	29	7205	59	341	38	7207	111	341	57
9	18055	137	5732	28	337	30	5743	57	337	58	5743	97	337	61
10	15032	90	7158	24	352	28	7173	52	352	61	7173	102	352	56

 $\Sigma si=40.0$ 

1	14156	100	6283	24	342	29	6296	53	341	47	6296	97	342	63
2	11164	75	5788	18	337	29	5797	44	338	120	5797	96	338	49
3	11166	86	5749	23	340	23	5756	86	340	40	5756	51	340	53
4	11166	86	5749	23	340	23	5756	86	340	40	5756	51	340	53
5	14521	138	6334	28	338	28	6346	55	338	55	6351	51	338	55
6	13527	85	6822	26	335	26	6836	46	335	67	6842	49	335	58
7	13528	76	6818	30	335	27	6833	47	335	56	6833	95	335	66
8	13149	94	6290	29	336	20	6313	62	336	55	6313	101	336	66
9	13526	96	7169	34	339	27	7183	51	339	60	7183	96	339	62
10	13146	86	6323	29	341	25	6340	47	341	66	6340	99	341	58

 $\Sigma si=50.0$ 

1	17174	82	8490	20	427	36	8493	46	427	42	8493	92	427	63
2	18492	75	9436	16	418	34	9437	101	418	70	9437	93	418	98
3	13189	81	7210	42	416	27	7220	55	416	87	7220	102	416	71
4	11395	83	5823	28	415	26	5829	42	415	59	5829	85	415	149
5	17028	117	7087	30	412	22	7094	46	412	72	7094	98	412	77
6	11765	81	5974	27	425	49	5978	52	425	60	5978	107	425	64
7	13858	99	6703	33	420	26	6714	90	420	67	6714	94	420	79
8	12293	67	6665	39	413	31	6674	87	413	73	6674	88	413	67
9	15352	73	7335	42	415	37	7341	58	415	63	7341	91	415	71
10	15356	79	7333	43	421	46	7344	89	421	61	7344	98	421	98

## STYLE 2, VARIATION 2(5X5X5, $\Sigma_{cap}=2.0$ )

S/N	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	15469	118	7583	36	422	46	7586	47	422	35	7611	77	422	51
2	15664	99	8024	23	412	24	8030	45	412	41	8054	63	412	54
3	19891	149	8700	26	415	47	8713	43	415	43	8725	68	415	98
4	17424	149	8278	33	417	28	8292	45	417	55	8308	54	417	51
5	12796	98	6129	32	423	25	6136	42	423	75	6159	57	423	45
6	16454	187	7037	18	410	24	7045	33	410	56	7057	63	410	120
7	12228	128	6152	35	419	29	6152	45	419	65	6153	72	420	88
8	10634	98	5505	41	411	15	5511	36	411	49	5536	65	411	52
9	12796	98	6129	32	423	25	6136	42	423	75	6159	57	423	45
10	15864	73	8021	21	417	33	8026	45	417	47	8048	65	417	61
<b><math>\Sigma_{si}=1.5</math></b>														
1	13756	76	6647	23	426	22	6659	30	426	65	6666	64	426	91
2	13658	130	6633	27	411	25	6646	34	411	100	6658	121	411	102
3	14821	116	7342	26	415	28	7356	38	415	120	7370	63	415	62
4	14125	103	6523	26	413	21	6535	40	413	54	6543	68	413	57
5	14083	141	6541	18	417	23	6551	42	417	45	6568	118	417	39
6	18204	152	8653	24	415	24	8665	42	415	42	8675	57	415	27
7	13658	130	6633	27	411	25	6646	34	411	100	6658	121	411	102
8	17055	127	8189	27	413	28	8200	40	413	70	8207	80	413	54
9	12892	101	6604	35	415	34	6611	40	415	62	6621	63	415	51
10	10634	98	5505	41	411	15	5511	36	411	49	5536	65	411	52
<b><math>\Sigma_{si}=2.0</math></b>														
1	13735	113	6765	23	423	25	6766	50	423	53	6768	60	423	47
2	13537	116	6941	21	431	31	6944	45	431	33	6952	57	431	47
3	13673	97	6675	22	419	23	6679	43	420	45	6683	60	420	44
4	13873	86	6565	35	412	31	6569	63	412	52	6573	65	412	50
5	12949	115	6657	42	415	26	6668	58	415	176	6680	127	415	79
6	12948	97	6657	21	412	39	6667	44	412	64	6680	53	412	45
7	13673	97	6675	22	419	23	6679	43	420	45	6683	60	420	44
8	16752	150	7494	20	417	30	7503	41	417	42	7507	62	417	49
9	14774	71	7612	45	419	30	7613	44	419	56	7625	67	419	45
10	16727	88	6545	24	413	32	6549	42	414	56	6552	65	414	48
<b><math>\Sigma_{si}=5.0</math></b>														
1	13023	101	6452	30	417	24	6454	54	417	53	6456	99	417	50
2	13646	99	6738	48	413	26	6744	37	413	78	6745	50	413	42
3	12652	86	6936	21	415	29	6938	45	415	45	6939	53	415	62
4	11489	101	6072	24	422	25	6076	44	422	98	6078	49	422	44
5	13023	101	6452	30	417	24	6454	54	417	53	6456	99	417	50
6	14202	113	6975	29	424	31	6983	66	424	54	6985	106	424	43
7	11319	93	5579	27	432	28	5581	45	432	78	5581	70	432	54
8	10906	99	5667	28	420	31	5673	62	420	54	5674	53	420	48
9	13343	92	6856	18	411	23	6869	38	411	48	6873	45	411	44
10	17015	98	8695	24	424	27	8698	58	424	54	8698	52	424	44



**$\Sigma si=10.0$** 

1	13158	111	6575	24	337	24	6598	43	337	42	6599	49	337	42
2	12850	113	6215	30	336	23	6226	56	336	66	6226	47	336	80
3	12850	113	6215	30	336	23	6226	56	336	66	6226	47	336	80
4	11160	113	5777	25	335	22	5789	30	335	45	5789	39	335	43
5	13531	110	7071	30	333	23	7085	54	333	65	7085	45	333	42
6	14161	159	6003	22	344	26	6014	62	344	52	6017	42	344	49
7	14152	122	6004	29	342	22	6011	60	342	65	6011	45	342	80
8	13522	120	6654	26	334	29	6673	54	334	58	6673	45	334	44
9	13514	114	6378	26	335	23	6391	60	335	52	6391	42	335	80
10	11169	120	5789	26	344	28	5797	55	344	58	5797	45	344	80

 **$\Sigma si=20.0$** 

1	13522	96	6866	21	339	22	6884	37	339	43	6884	43	339	43
2	13514	146	6176	26	338	24	6186	68	338	58	6186	61	338	42
3	14516	147	6294	26	338	26	6307	35	338	41	6307	60	338	45
4	14518	145	6298	26	337	25	6312	35	337	48	6312	60	337	42
5	12850	109	6080	21	336	20	6088	35	336	45	6088	61	336	45
6	12848	109	6078	26	335	24	6087	37	335	45	6087	61	335	39
7	13515	141	6162	28	336	20	6175	38	336	38	6186	53	336	41
8	13515	141	6162	28	336	20	6175	38	336	38	6186	53	336	41
9	15839	167	6686	22	346	21	6705	70	346	40	6705	48	346	40
10	12499	105	6163	22	346	20	6177	35	346	38	6179	61	346	42

 **$\Sigma si=30.0$** 

1	13136	139	6132	28	337	28	6144	36	337	43	6144	37	337	44
2	13152	128	6212	25	337	34	6234	45	337	49	6234	48	337	47
3	14157	119	6567	22	338	23	6587	36	338	41	6587	47	338	36
4	13143	112	6259	24	335	32	6283	42	335	41	6283	46	335	49
5	13143	112	6259	24	335	32	6283	42	335	41	6283	46	335	49
6	13144	110	6239	26	334	33	6259	42	334	42	6259	44	334	40
7	13518	110	6410	26	334	33	6431	36	334	42	6431	48	334	49
8	13157	112	7191	22	341	32	7205	45	341	41	7207	46	341	40
9	18055	193	5732	24	337	23	5743	39	337	40	5743	47	337	43
10	15032	125	7158	24	352	33	7173	38	352	42	7173	48	352	40

 **$\Sigma si=40.0$** 

1	14156	112	6283	28	342	22	6296	54	341	52	6296	53	342	42
2	11164	109	5788	27	337	31	5797	44	338	37	5797	37	338	39
3	11166	125	5749	18	340	26	5756	42	340	51	5756	48	340	63
4	11166	125	5749	18	340	26	5756	42	340	51	5756	48	340	63
5	14521	162	6334	18	338	32	6346	44	338	52	6351	46	338	48
6	13525	100	6817	23	335	33	6830	46	335	67	6842	88	335	44
7	13528	83	6818	27	335	26	6833	52	335	52	6833	61	335	43
8	13149	141	6290	47	336	30	6313	48	336	56	6313	45	336	59
9	13526	99	7169	25	339	23	7183	40	339	56	7183	45	339	49
10	13146	123	6323	27	341	32	6340	44	341	52	6340	48	341	65

 **$\Sigma si=50.0$** 

1	17174	92	8490	10	427	17	8493	33	427	36	8493	51	427	38
2	18492	95	9436	24	418	24	9437	38	418	45	9437	41	418	76
3	13189	102	7210	26	416	30	7220	39	416	49	7220	37	416	51
4	11395	80	5823	23	415	28	5829	40	415	44	5829	91	415	60
5	17028	127	7087	23	412	23	7094	56	412	34	7094	47	412	55
6	11765	124	5974	22	425	30	5978	32	425	41	5978	47	425	45
7	13858	120	6703	15	420	20	6714	37	420	25	6714	44	420	39
8	12293	85	6665	24	413	21	6674	36	413	22	6674	35	413	45
9	15352	93	7335	30	415	25	7341	65	415	48	7341	81	415	53
10	15356	92	7333	30	421	29	7344	69	421	43	7344	79	421	43

## STYLE 1, VARIATION 1(5X5X5, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	8049	85	8048	27	3306	33	8049	146	3306	29	8049	170	3306	40	3306	31
2	8222	51	8222	62	3161	37	8222	106	3161	22	8222	104	3161	29	3161	65
3	9535	137	7674	34	4504	21	7674	181	4504	36	7674	81	4504	25	4504	33
4	8491	110	8215	27	3262	37	8215	151	3262	41	8215	181	3262	29	3262	72
5	5839	60	5839	57	3528	28	5839	97	3528	33	5839	156	3528	32	3528	20
6	7816	62	7816	26	3791	25	7816	114	3791	26	7816	265	3971	28	3791	34
7	5907	56	5907	34	2334	29	5907	60	2334	33	5907	93	2334	37	2334	34
8	5502	75	5502	80	4019	32	5502	90	4019	46	5502	214	4019	38	4019	23
9	5839	60	5839	57	3528	28	5839	97	3528	33	5839	156	3528	32	3528	20
10	8110	76	8110	32	4281	47	8110	144	4281	27	8110	236	4281	37	4281	21
<b><math>\Sigma_{si}=1.5</math></b>																
1	6312	70	6312	35	2738	22	6312	137	2738	29	6312	84	2738	30	2738	31
2	6805	69	6805	20	2968	49	6805	133	2968	58	6805	97	2968	36	2968	42
3	6935	85	6935	31	3856	49	6935	77	3856	27	6935	254	3856	35	3856	84
4	6009	58	6009	25	4894	45	6009	99	4894	44	6009	157	4894	43	4894	74
5	6660	69	6660	22	4143	23	6660	140	4143	22	6660	180	4143	38	4143	23
6	8570	52	8570	29	3937	49	8570	79	3937	37	8570	236	3937	46	3937	28
7	6805	69	6805	20	2968	49	6805	133	2968	58	6805	97	2968	36	2968	42
8	7491	80	7491	26	4846	24	7491	138	4846	25	7491	188	4846	28	4846	26
9	7388	116	6887	50	3842	45	6887	88	3842	34	6887	104	3842	50	3842	16
10	5502	75	5502	80	4019	32	5502	90	4019	46	5502	214	4019	38	4019	23
<b><math>\Sigma_{si}=2.0</math></b>																
1	6733	70	6733	29	5037	54	6733	80	5037	33	6733	171	5037	34	5037	23
2	6908	87	6907	49	4213	26	6908	108	4213	27	6908	197	4213	25	4213	29
3	6049	53	6049	22	2835	50	6049	72	2835	32	6049	116	2835	44	2835	22
4	7088	60	7088	23	4368	18	7088	140	4368	30	7088	140	4368	42	4368	22
5	7285	113	6548	49	5108	50	6548	140	5108	34	6548	93	5108	40	5108	23
6	5479	45	5479	17	2926	25	5479	79	2926	28	5479	98	2926	29	2926	22
7	6049	53	6049	22	2835	50	6049	72	2835	32	6049	116	2835	44	2835	22
8	7479	48	7479	23	4666	42	7479	75	4666	25	7479	77	4666	31	4666	14
9	7598	56	7598	64	3209	22	7598	162	3209	30	7598	155	3209	43	3209	15
10	7333	52	7333	27	2828	49	7333	62	2828	27	7333	101	2828	54	2828	22
<b><math>\Sigma_{si}=5.0</math></b>																
1	6332	46	6332	20	4483	22	6332	73	4483	29	6332	130	4483	22	4483	23
2	6374	37	6374	17	2752	22	6374	72	2752	31	6374	69	2752	22	2752	17
3	5944	38	5944	12	2741	40	5944	83	2741	26	5944	124	2741	24	2741	20
4	5554	45	5554	20	4200	27	5554	56	4200	20	5554	146	4200	22	4200	20
5	6332	46	6332	20	4483	22	6332	73	4483	29	6332	130	4483	22	4483	23
6	7569	42	7569	21	2470	36	7569	115	2470	29	7569	87	2470	29	2470	13
7	6264	58	6264	15	3602	48	6264	67	3602	24	6264	66	3602	29	3602	17
8	5743	73	5711	27	3698	49	5711	90	3698	41	5711	110	3698	21	3698	20
9	5831	50	5831	21	2897	20	5831	44	2897	26	5831	57	2897	26	2897	21
10	8672	58	8672	20	4362	24	8672	80	4362	30	8672	88	4362	35	4362	16



**$\Sigma si=10.0$** 

1	5582	25	5582	15	2985	34	5582	61	2985	24	5582	65	2985	21	2985	18
2	7952	43	7592	21	3206	29	7592	113	3206	27	7592	79	3206	23	3206	20
3	5574	32	5574	14	4400	30	5574	71	4400	41	5574	60	4400	48	4400	14
4	5960	32	5960	14	4198	31	5960	43	4198	35	5960	84	4198	28	4198	20
5	5572	41	5572	22	4539	20	5572	53	4539	18	5572	32	4539	20	4539	16
6	5593	49	5593	20	3955	37	5593	66	3955	25	5593	154	3955	25	3955	21
7	5964	34	5964	16	2929	50	5964	64	2929	26	5964	76	2929	20	2929	22
8	5964	34	5964	16	2929	50	5964	64	2929	26	5964	76	2929	20	2929	22
9	5581	31	5581	19	4226	54	5581	48	4226	45	5581	63	4226	32	4226	15
10	5957	33	5957	19	3403	61	5957	74	3403	53	5957	55	3403	26	3403	12

 **$\Sigma si=20.0$** 

1	5577	34	5577	14	3499	42	5577	123	3499	35	5577	70	3499	26	3499	14
2	5576	30	5576	22	3645	33	5576	72	3645	27	5576	71	3645	24	3645	14
3	5576	30	5576	22	3645	33	5576	72	3645	27	5576	71	3645	24	3645	14
4	5574	39	5574	14	3953	36	5574	44	3953	41	5574	59	3953	34	3953	15
5	5574	30	5574	17	3953	42	5574	72	3953	41	5574	71	3953	24	3953	15
6	5578	34	5578	17	3386	36	5578	82	3386	24	5578	69	3386	30	3386	20
7	5595	40	5595	20	3383	19	5595	45	3383	21	5595	59	3383	24	3383	20
8	5969	51	5969	16	3502	27	5969	68	3502	57	5969	55	3502	23	3502	19
9	7293	44	7293	22	3441	39	7293	101	3441	29	7293	96	3441	22	3441	22
10	7948	77	7364	21	3202	41	7364	58	3202	27	7364	128	3202	23	3202	18

 **$\Sigma si=30.0$** 

1	5959	35	5959	12	3861	31	5959	108	3861	26	5959	83	3861	26	3861	12
2	5575	35	5575	15	3517	37	5575	69	3517	24	5575	47	3517	21	3517	19
3	7941	85	5968	16	2919	36	5968	59	2919	24	5968	55	2919	27	2919	21
4	5575	35	5575	14	4301	48	5575	60	4300	26	5575	58	4300	22	4300	15
5	8939	127	7354	34	3268	42	7354	67	3268	18	7354	57	3268	25	3268	15
6	7941	65	6015	14	3103	36	6015	97	3103	26	6015	60	3103	33	3103	19
7	5575	35	5575	15	3517	37	5575	69	3517	24	5575	47	3517	21	3517	19
8	5959	36	5959	14	4042	18	5959	69	4042	19	5959	112	4042	24	4042	17
9	7945	89	5687	14	3979	40	5687	141	3979	24	5687	96	3979	27	3979	19
10	5951	30	5951	14	3491	35	5951	59	3491	19	5951	70	3491	31	3491	19

 **$\Sigma si=40.0$** 

1	5580	29	5580	16	2741	42	5580	60	2741	37	5580	51	2741	24	2741	16
2	5576	39	5576	14	3667	39	5576	95	3667	24	5576	89	3667	22	3667	14
3	5574	35	5574	14	3516	30	5574	112	3516	41	5574	85	3516	31	3516	16
4	5574	30	5574	14	3516	30	5574	85	3516	30	5574	82	3516	24	3516	16
5	5610	65	5610	16	3815	40	5610	43	3815	22	5610	36	3815	39	3815	20
6	7944	57	7944	20	3011	44	7944	49	3011	21	7944	95	3011	22	3011	20
7	5572	33	5572	14	4139	18	5572	75	4139	24	5572	65	4139	25	4139	15
8	5580	30	5580	22	2927	20	5580	49	2927	18	5580	67	2927	22	2927	17
9	5577	39	5577	22	3921	32	5577	76	3921	27	5577	76	3921	21	3921	14
10	7942	37	7942	18	3027	38	7942	65	3027	24	7942	59	3027	21	3027	18

 **$\Sigma si=50.0$** 

1	5688	43	5688	27	2330	49	5688	102	2330	30	5688	53	2330	24	2330	20
2	9329	45	9329	16	3707	35	9329	42	3707	22	9329	69	3707	21	3707	18
3	6921	47	6921	22	2729	51	6921	105	2729	25	6921	97	2729	24	2729	19
4	7360	38	7360	16	3269	37	7360	81	3269	30	7360	60	3269	26	3269	17
5	7736	48	7736	14	4460	43	7736	52	4460	20	7736	56	4460	22	4460	18
6	5633	50	5633	18	3258	41	5633	60	3258	20	5633	52	3258	24	3258	17
7	6003	45	6003	16	3197	45	6003	60	3197	20	6003	56	3197	24	3197	18
8	7377	52	7202	18	3761	44	7202	65	3761	22	7202	53	3761	24	3761	17
9	6396	52	6396	18	3047	41	6396	60	3047	21	6396	56	3047	22	3047	18
10	7360	38	7360	16	3269	37	7360	81	3269	30	7360	60	3269	26	3269	17

## STYLE 2, VARIATION 1(5X5X5, $\Sigma_{cap}=6.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	8028	49	8028	13	3285	59	8028	43	3285	28	8049	126	3285	44	3285	15
2	8197	76	8197	12	3136	38	8197	31	3136	35	8222	79	3136	44	3136	11
3	9526	102	7660	13	4485	18	7660	34	4485	40	7670	68	4485	45	4485	11
4	8478	85	8202	45	3250	26	8202	66	3250	26	8214	41	3250	42	3250	12
5	5815	70	5815	11	3509	33	5815	49	3509	47	5839	50	3509	31	3509	27
6	7807	87	7807	10	3782	29	7807	29	3782	34	7816	50	3782	43	3782	10
7	5898	84	5898	9	2325	20	5898	78	2325	39	5907	64	2325	40	2325	9
8	5495	77	5495	12	4006	49	5495	32	4006	58	5502	58	4006	40	4006	15
9	5815	70	5815	11	3509	33	5815	49	3509	47	5839	50	3509	31	3509	27
10	8089	61	8089	11	4255	25	8089	68	4255	23	8110	106	4255	33	4255	36
<b><math>\Sigma_{si}=1.5</math></b>																
1	6309	43	6309	12	2716	25	6309	51	2716	24	6312	83	2716	43	2716	10
2	6800	85	6800	12	2961	35	6800	49	2961	29	6805	113	2961	33	2961	14
3	6925	65	6925	35	3846	29	6925	71	3846	28	6935	101	3846	28	3846	14
4	6000	79	6000	9	4885	32	6000	31	4885	41	6009	90	4885	42	4885	9
5	6652	38	6652	22	4125	23	6652	37	4125	37	6660	127	4125	37	4125	12
6	8561	45	8561	14	3932	28	8561	83	3932	65	8570	95	3932	49	3932	15
7	6800	85	6800	12	2961	35	6800	49	2961	29	6805	113	2961	33	2961	14
8	7480	53	7480	45	4835	32	7480	77	4835	37	7491	98	4835	35	4835	39
9	7382	97	6885	11	3836	41	6885	76	3836	33	6886	122	3836	40	3836	15
10	5495	77	5495	12	4006	49	5495	32	4006	58	5502	58	4006	40	40026	15
<b><math>\Sigma_{si}=2.0</math></b>																
1	6731	82	6731	44	5035	74	6731	36	5035	34	6733	39	5035	36	5035	33
2	6900	50	6900	29	4209	31	6900	57	4209	28	6908	100	4209	45	4209	31
3	6047	76	6047	13	2833	32	6047	58	2833	27	6049	56	2833	31	2833	12
4	7068	74	7068	12	4364	27	7068	97	4364	28	7088	95	4364	43	4364	15
5	7277	98	6533	15	5106	35	6533	73	5106	26	6546	96	5106	41	5106	17
6	5467	85	5467	11	2914	21	5467	29	2914	34	5479	54	2914	41	2914	31
7	6047	76	6047	13	2833	32	6047	58	2833	27	6049	56	2833	31	2833	12
8	7478	90	7478	11	4655	24	7478	97	4655	28	7479	106	4655	34	4655	15
9	7590	66	7590	11	3195	28	7590	45	3195	38	7598	92	3195	37	3195	17
10	7332	76	7332	32	2828	27	7332	29	2828	44	7333	82	2828	40	2828	11
<b><math>\Sigma_{si}=5.0</math></b>																
1	6324	71	6324	17	4482	31	6324	46	4482	33	6324	56	4482	30	4482	13
2	6374	52	6374	31	2752	27	6374	72	2752	26	6374	53	2752	26	2752	34
3	5940	38	5940	14	2741	30	5940	43	2741	32	5944	66	2741	34	2741	16
4	5550	55	5550	43	4196	29	5550	66	4196	30	5554	90	4196	34	4196	12
5	6324	71	6324	17	4482	31	6324	46	4482	33	6324	56	4482	30	4482	13
6	7569	50	7569	13	2470	22	7569	70	2470	36	7569	49	2470	40	2470	11
7	6264	59	6264	12	3602	43	6264	32	3602	25	6264	40	3602	33	3602	12
8	5743	48	5709	11	3698	22	5709	80	3698	63	5711	78	3698	27	3698	31
9	5823	79	5823	10	2888	16	5823	10	2888	41	5831	58	2888	51	2888	16
10	8672	76	8672	34	4362	36	8672	69	4362	51	8672	54	4362	29	4362	16

$\Sigma si=10.0$ 

1	5582	68	5582	12	2979	25	5582	72	2979	35	5582	73	2979	26	2979	14
2	7952	125	7592	17	3206	25	7592	70	3206	27	7592	69	3206	30	3206	14
3	5574	53	5574	12	4400	39	5574	48	4400	29	5574	66	4400	29	4400	10
4	5960	42	5960	14	4198	39	5960	36	4198	27	5960	76	4198	25	4198	14
5	5572	47	5572	13	4539	74	5572	29	4539	45	5572	84	4539	33	4539	29
6	5580	54	5580	12	3955	59	5580	41	3955	28	5593	49	3955	24	3955	16
7	5964	43	5964	14	2929	37	5964	43	2929	43	5964	53	2929	41	2929	12
8	5964	43	5964	14	2929	37	5964	43	2929	43	5964	53	2929	41	2929	12
9	5581	41	5581	12	4226	36	5581	59	4226	37	5581	62	4226	28	4226	17
10	5957	40	5957	14	3403	48	5957	46	3403	28	5957	71	3403	25	3403	14

 $\Sigma si=20.0$ 

1	5577	82	5577	12	3499	58	5577	45	3499	32	5577	92	3499	24	3499	10
2	5576	41	5576	19	3645	28	5576	35	3645	47	5576	60	3645	63	3645	14
3	5576	41	5576	19	3645	28	5576	35	3645	47	5576	60	3645	63	3645	14
4	5574	50	5574	20	3953	48	5574	52	3953	45	5574	68	3953	42	3953	18
5	5574	41	5574	19	3953	28	5574	35	3953	45	5574	62	3953	40	3953	14
6	5578	45	5578	22	3386	34	5578	33	3386	30	5578	60	3386	59	3386	17
7	5595	42	5595	19	3383	42	5595	35	3383	45	5595	60	3383	40	3383	17
8	5969	59	5969	14	3502	21	5969	33	3502	42	5969	60	3502	40	3502	19
9	7293	69	7293	21	3441	28	7293	53	3441	42	7293	60	3441	52	3441	17
10	7939	84	7364	17	3202	25	7364	90	3202	54	7364	69	3202	63	3202	13

 $\Sigma si=30.0$ 

1	5959	53	5959	16	3861	29	5959	83	3861	55	5959	85	3861	69	3861	16
2	5575	50	5575	17	3517	25	5575	50	3517	28	5575	68	3517	51	3517	16
3	7941	98	5968	22	2919	34	5968	78	2919	27	5968	60	2919	61	2919	18
4	5575	88	5575	20	4301	37	5575	39	4300	49	5575	117	4300	52	4300	18
5	8939	141	7354	16	3268	36	7354	42	3268	28	7354	62	3268	55	3268	16
6	7941	86	6015	18	3103	30	6015	48	3103	27	6015	68	3103	59	3103	18
7	5575	50	5575	17	3517	25	5575	50	3517	28	5575	68	3517	51	3517	16
8	5959	88	5959	22	4042	30	5959	59	4042	27	5959	82	4042	55	4042	17
9	7945	52	5687	18	3979	34	5687	42	3979	27	5687	68	3979	52	3979	18
10	5951	88	5951	18	3491	35	5951	50	3491	27	5951	68	3491	55	3491	18

 $\Sigma si=40.0$ 

1	5580	48	5580	17	2741	32	5580	45	2741	43	5580	58	2741	48	2741	17
2	5576	56	5576	20	3667	28	5576	51	3667	30	5576	42	3667	52	3667	15
3	5574	55	5574	20	3516	28	5574	35	3516	46	5574	57	3516	40	3516	16
4	5574	49	5574	20	3516	28	5574	40	3516	30	5574	45	3516	52	3516	16
5	5610	89	5610	20	3815	28	5610	43	3815	30	5610	52	3815	48	3815	16
6	7944	36	7944	16	3011	24	7944	54	3011	58	7944	46	3011	70	3011	13
7	5572	56	5572	20	4139	41	5572	82	4139	34	5572	46	4139	51	4139	12
8	5580	52	5580	20	2927	38	5580	57	2927	35	5580	52	2927	60	2927	30
9	5577	52	5577	20	3921	28	5577	43	3921	32	5577	46	3921	52	3921	16
10	7942	52	7942	20	3027	28	7942	43	3027	35	7942	46	3027	60	3027	16

 $\Sigma si=50.0$ 

1	5688	63	5688	12	2330	19	5688	39	2330	39	5688	43	2330	50	2330	13
2	9329	56	9329	16	3707	30	9329	74	3707	48	9329	79	3707	52	3707	11
3	6921	45	6921	18	2729	40	6921	33	2729	39	6921	59	2729	34	2729	18
4	7360	64	7360	14	3269	24	7360	78	3269	47	7360	43	3269	71	3269	7
5	7736	65	7736	16	4460	24	7736	35	4460	40	7736	45	4460	52	4460	11
6	5633	66	5633	18	3258	25	5633	77	3258	40	5633	43	3258	50	3258	18
7	6003	55	6003	18	3197	31	6003	41	3197	37	6003	45	3197	46	3197	18
8	7377	69	7202	14	3761	29	7202	25	3761	58	7202	53	3761	57	3761	12
9	6396	78	6396	20	3047	32	6396	43	3047	21	6396	91	3047	45	3047	11
10	7360	64	7360	14	3269	24	7360	78	3269	47	7360	43	3269	71	3269	7

## STYLE 1, VARIATION 2(5X5X5, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	8049	139	7176	54	426	49	7177	57	426	72	7182	98	426	72
2	8222	96	7999	77	424	45	8003	62	424	54	8007	192	424	60
3	9535	183	6860	36	417	46	6864	184	417	85	6866	171	417	78
4	8491	174	7924	30	419	39	7926	99	419	173	7927	186	419	78
5	5839	67	5839	45	422	46	5839	93	422	81	5839	233	422	162
6	7816	90	6709	50	412	40	6714	80	412	78	6720	208	412	87
7	5907	81	5907	58	417	46	5907	66	417	93	5907	124	417	96
8	5502	62	5502	75	419	45	5502	53	419	89	5502	133	419	93
9	5839	67	5839	45	422	46	5839	93	422	81	5839	233	422	162
10	8110	84	6708	53	429	37	6714	66	429	74	6721	191	429	74
<b><math>\Sigma_{si}=1.5</math></b>														
1	6312	86	6312	28	429	30	6312	51	429	58	6312	94	429	69
2	6805	83	6696	30	415	46	6702	83	415	67	6705	85	415	78
3	6935	74	6935	32	414	65	6935	49	414	139	6935	71	414	51
4	6009	93	5599	24	411	28	5606	100	411	66	5607	82	411	58
5	6660	100	5870	69	417	36	5873	69	417	162	5876	88	417	69
6	8570	122	8005	34	420	61	8009	76	420	100	8015	95	420	93
7	6805	83	6696	30	415	46	6702	83	415	67	6705	85	415	78
8	7491	79	7491	24	413	33	7491	74	412	77	7491	96	413	58
9	7388	127	5996	30	416	46	5999	96	416	100	6002	106	416	80
10	5502	62	5502	75	419	45	5502	53	419	89	5502	133	419	93
<b><math>\Sigma_{si}=2.0</math></b>														
1	6733	79	6733	41	424	37	6733	77	424	78	6733	81	424	72
2	6908	73	6907	23	431	28	6908	66	431	52	6908	78	431	81
3	6049	72	6049	47	418	28	6049	80	418	70	6049	103	418	64
4	7088	82	6314	29	410	27	6321	71	410	112	6328	115	410	74
5	7285	129	5874	14	417	27	5879	91	417	108	5880	94	417	126
6	5479	54	5479	50	411	50	5479	79	411	70	5479	86	411	64
7	6049	72	6049	47	418	28	6049	80	418	70	6049	103	418	64
8	7479	85	6655	29	416	32	6660	167	416	71	6663	158	416	86
9	7598	56	7598	47	414	41	7598	75	414	95	7598	89	414	66
10	7333	61	7333	27	417	54	7333	100	417	79	7333	68	417	123
<b><math>\Sigma_{si}=5.0</math></b>														
1	6332	40	6332	16	414	26	6332	61	414	54	6332	78	414	70
2	6374	54	5889	27	409	30	5894	48	408	102	5894	53	409	42
3	5944	42	5944	12	412	20	5944	62	412	63	5944	108	412	80
4	5554	41	5554	16	419	34	5554	61	419	57	5554	101	419	46
5	6332	40	6332	16	414	26	6332	61	414	54	6332	78	414	70
6	7569	69	7312	23	422	26	7316	48	422	63	7317	132	422	79
7	6264	66	6090	42	425	32	6093	50	425	66	6093	55	425	73
8	5743	69	5608	28	408	19	5609	63	408	42	5608	77	408	62
9	5831	51	5831	19	412	21	5831	47	412	40	5831	46	412	59
10	8672	61	8672	40	419	22	8672	36	419	65	8672	35	419	64

$\Sigma si=10.0$ 

1	5582	39	5582	13	342	26	5582	74	342	100	5582	93	342	66
2	7952	72	6065	41	337	26	6084	55	337	57	6084	96	337	64
3	5574	37	5574	13	335	23	5574	49	335	48	5574	80	335	48
4	5960	40	5788	32	332	23	5797	54	332	63	5797	53	332	64
5	5572	32	5572	13	332	23	5572	62	332	92	5572	51	332	65
6	5593	47	5593	37	335	23	5593	86	335	43	5593	165	335	49
7	5964	39	5731	17	338	27	5738	74	338	51	5738	65	338	115
8	5964	39	5731	17	338	27	5738	74	338	51	5738	65	338	115
9	5581	38	5581	32	334	23	5581	45	334	43	5581	69	334	82
10	5957	47	5774	18	336	23	5781	73	336	46	5781	71	336	54

 $\Sigma si=20.0$ 

1	5577	38	5577	13	337	23	5577	109	337	50	5577	65	337	61
2	5576	31	5576	13	337	22	5576	82	337	56	5576	62	337	49
3	5576	31	5576	13	337	22	5576	82	337	56	5576	62	337	49
4	5574	31	5574	13	335	23	5574	68	335	39	5574	60	335	54
5	5574	30	5574	13	335	22	5574	62	335	50	5574	60	335	54
6	5578	37	5578	32	336	23	5578	46	336	47	5578	61	336	66
7	5595	50	5595	42	345	27	5595	67	345	46	5595	70	345	66
8	5969	69	5828	41	339	25	5836	55	339	46	5838	59	339	62
9	7293	55	6044	32	340	26	6052	59	340	47	6052	95	340	73
10	7948	113	5761	28	335	22	5768	81	335	55	5770	51	335	53

 $\Sigma si=30.0$ 

1	5959	35	5959	12	3861	41	5800	19	337	23	5805	66	337	67
2	5575	30	5575	13	337	23	5575	75	337	52	5575	65	337	54
3	7941	104	5599	22	337	23	5602	65	337	50	5602	55	337	51
4	5575	40	5575	13	335	23	5575	71	335	39	5575	72	335	61
5	8939	68	5666	24	335	23	5673	67	335	39	5673	53	335	62
6	7941	110	5615	19	334	23	5618	66	334	92	5618	61	334	67
7	5575	30	5575	13	337	23	5575	75	337	52	5575	65	337	54
8	5959	40	5747	32	334	23	5756	57	334	19	5756	87	334	67
9	7945	136	5587	21	336	23	5588	88	336	104	5588	52	336	57
10	5951	43	5649	31	337	23	5654	50	337	52	5654	53	337	47

 $\Sigma si=40.0$ 

1	5580	39	5580	32	339	28	5580	49	339	47	5580	74	339	71
2	5576	35	5576	13	338	22	5576	76	338	55	5576	61	338	54
3	5574	35	5574	13	338	25	5574	87	338	38	5574	70	338	51
4	5574	35	5574	13	338	22	5574	75	338	45	5574	72	338	50
5	5610	54	5610	18	335	22	5610	54	335	57	5610	50	335	54
6	7944	89	6111	45	335	22	6120	57	335	53	6125	40	335	63
7	5572	35	5572	13	335	23	5572	66	335	41	5572	68	335	61
8	5580	44	5580	13	336	22	5580	68	336	97	5580	70	336	59
9	5577	37	5577	13	335	23	5577	63	335	43	5577	77	335	63
10	7942	64	5738	17	338	23	5747	101	338	67	5747	51	338	48

 $\Sigma si=50.0$ 

1	5688	42	5688	37	422	47	5688	50	422	47	5688	73	422	61
2	9329	78	8806	16	419	32	8814	67	419	63	8814	59	419	57
3	6921	66	6865	50	425	34	6869	62	425	69	6869	106	425	66
4	7360	51	7091	26	412	49	7094	87	412	97	7094	59	412	92
5	7736	48	7736	13	404	21	7736	48	404	58	7736	93	404	52
6	5633	45	5633	35	408	20	5633	92	408	71	5633	67	408	76
7	6003	48	6003	30	410	21	6003	67	410	64	6003	59	410	65
8	7377	48	6086	30	415	20	6095	88	415	65	6095	67	415	65
9	6396	51	6396	28	411	24	6396	62	411	63	6396	80	411	66
10	7360	51	7091	26	412	49	7094	87	412	97	7094	59	412	92

## STYLE 1, VARIATION 1(5X5X5, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	8028	64	7157	31	410	31	7158	37	410	47	7182	60	410	48
2	8197	47	7980	19	412	28	7984	40	412	40	8007	46	412	52
3	9526	101	6850	21	414	52	6853	33	414	44	6866	62	414	109
4	8478	73	7912	22	414	49	7914	40	414	33	7927	47	414	67
5	5815	73	5815	52	421	51	5815	94	421	55	5839	53	421	92
6	7807	101	7807	22	412	48	7807	33	412	34	7816	62	412	97
7	5898	47	5898	19	411	28	5898	64	411	47	5907	85	411	58
8	5475	43	5475	13	409	45	5475	72	409	36	5502	44	409	44
9	5815	73	5815	52	421	51	5815	94	421	55	5839	53	421	92
10	8089	35	6701	19	417	39	6707	30	417	23	6721	57	417	64
<b><math>\Sigma_{si}=1.5</math></b>														
1	6309	35	6309	14	426	51	6309	62	426	55	6312	57	426	85
2	6800	36	6694	21	411	26	6691	80	411	41	6705	51	411	41
3	6925	63	6925	10	412	48	6925	33	412	39	6935	52	412	83
4	6000	60	5593	14	406	43	5599	42	406	35	5607	50	406	74
5	6652	52	5857	24	417	44	5860	35	417	47	5876	72	417	41
6	8561	103	7999	16	415	38	8004	31	415	38	8015	49	415	64
7	6800	36	6694	21	411	26	6691	80	411	41	6705	51	411	41
8	7480	57	7480	18	412	36	7480	30	412	52	7491	39	412	66
9	7382	69	5994	15	414	43	5997	38	414	49	6002	58	414	89
10	5475	43	5475	13	409	45	5475	72	409	36	5502	44	409	44
<b><math>\Sigma_{si}=2.0</math></b>														
1	6731	33	6731	17	416	43	6731	44	416	37	6733	45	416	76
2	6900	36	6900	13	431	46	6900	39	431	31	6908	64	431	78
3	6047	37	6047	12	416	39	6047	43	416	37	6049	48	416	37
4	7068	42	6310	18	405	22	6317	41	405	29	6328	55	405	88
5	7277	87	5861	19	416	45	5865	37	416	30	5880	49	416	42
6	5467	58	5467	16	411	43	5467	23	411	30	5479	71	411	87
7	6047	37	6047	12	416	39	6047	43	416	37	6049	48	416	37
8	7478	38	6652	22	414	48	6657	27	414	33	6663	59	414	81
9	7590	46	7590	18	414	30	7590	47	414	47	7598	48	414	42
10	7332	48	7332	22	417	52	7332	55	417	39	7332	56	417	55
<b><math>\Sigma_{si}=5.0</math></b>														
1	6324	21	6324	14	413	21	6324	70	413	26	6332	51	413	78
2	6374	30	5889	26	409	49	5894	35	408	49	5894	46	409	69
3	5940	24	5940	25	410	42	5940	41	410	54	5944	47	410	78
4	5550	36	5550	22	419	38	5550	38	419	53	5554	44	419	73
5	6324	21	6324	14	413	21	6324	70	413	26	6332	51	413	78
6	7569	67	7312	26	422	28	7316	43	422	38	7317	122	422	45
7	6264	30	6090	28	425	44	6093	41	425	44	6093	49	425	47
8	5743	68	5605	29	407	58	5606	36	407	32	5608	50	407	93
9	5823	57	5823	17	408	42	5823	37	408	28	5831	62	408	79
10	8672	67	8672	42	419	45	8672	70	419	82	8672	45	419	84



$\Sigma si=10.0$ 

1	5582	62	5582	46	342	43	5582	36	342	31	5582	66	342	66
2	7952	78	6065	21	3206	29	337	21	6084	40	6084	41	337	62
3	5574	37	5574	14	335	37	5574	43	335	57	5574	77	335	67
4	5960	42	5788	16	332	16	5797	30	332	50	5797	44	332	63
5	5572	42	5572	22	332	29	5572	40	332	64	5572	40	332	74
6	5580	37	5580	14	335	37	5580	41	335	64	5593	96	335	66
7	5964	54	5731	19	338	19	5738	93	338	31	5738	51	338	37
8	5964	54	5731	19	338	19	5738	93	338	31	5738	51	338	37
9	5581	37	5581	19	334	38	5581	42	334	58	5581	51	334	67
10	5957	54	5734	19	336	18	5781	35	336	25	5781	39	336	44

 $\Sigma si=20.0$ 

1	5577	65	5577	17	337	47	5577	34	337	63	5577	30	337	80
2	5576	46	5576	20	337	20	5576	33	337	26	5576	47	337	51
3	5576	46	5576	20	337	20	5576	33	337	26	5576	47	337	51
4	5574	34	5574	14	335	18	5574	33	335	16	5574	44	335	41
5	5574	40	5574	18	335	18	5574	33	335	18	5574	42	335	50
6	5578	37	5578	14	336	22	5578	47	336	16	5578	41	336	36
7	5595	37	5595	14	345	18	5595	33	345	16	5595	47	345	41
8	5969	55	5828	18	339	22	5836	33	339	28	5838	44	339	36
9	7293	57	6044	28	340	24	6052	29	340	50	6052	40	340	56
10	7939	78	5721	20	335	18	5729	31	335	36	5770	82	335	41

 $\Sigma si=30.0$ 

1	5959	44	5800	23	337	23	5805	37	337	50	5805	36	337	51
2	5575	37	5575	20	337	27	5575	34	337	35	5575	47	337	45
3	7941	141	5599	16	337	22	5602	30	337	15	5602	43	337	44
4	5575	37	5575	14	335	18	5575	35	335	33	5575	45	335	52
5	8939	190	5666	16	335	18	5673	40	335	39	5673	47	335	55
6	7941	125	5615	14	334	23	5618	35	334	32	5618	44	334	52
7	5575	37	5575	20	337	27	5575	34	337	35	5575	47	337	45
8	5959	40	5747	16	334	23	5756	35	334	15	5756	43	334	52
9	7945	136	5587	23	336	27	5588	35	336	32	5588	44	336	51
10	5951	59	5649	18	337	23	5654	35	337	33	5654	45	337	52

 $\Sigma si=40.0$ 

1	5580	47	5580	17	335	18	5580	50	335	18	5580	42	335	38
2	5576	35	5576	19	338	18	5576	41	338	15	5576	41	338	37
3	5574	36	5574	32	338	18	5574	32	338	25	5574	52	338	37
4	5574	35	5574	22	338	18	5574	32	338	25	5574	45	338	37
5	5610	35	5610	22	335	18	5610	40	335	25	5610	42	335	37
6	7944	67	6111	24	335	18	6120	32	335	25	6125	45	335	38
7	5572	35	5572	14	335	18	5572	44	335	15	5572	50	335	44
8	5580	62	5580	20	336	29	5580	46	336	43	5580	41	336	41
9	5577	37	5577	22	335	18	5577	32	335	25	5577	42	335	38
10	7942	52	5738	20	338	18	5747	44	338	28	5747	42	338	41

 $\Sigma si=50.0$ 

1	5688	49	5688	20	422	23	5688	40	422	35	5688	45	422	60
2	9329	65	8806	21	419	16	8814	35	419	55	8814	34	419	58
3	6921	43	6865	25	425	27	6869	42	425	45	6869	45	425	58
4	7360	43	7091	21	412	19	7094	87	412	35	7094	50	412	47
5	7736	45	7736	20	404	23	7736	45	404	45	7736	34	404	60
6	5633	62	5633	20	408	25	5633	35	408	45	5633	35	408	58
7	6003	51	6003	26	410	32	6003	28	410	64	6003	55	410	70
8	7377	106	6086	27	415	24	6095	48	415	33	6095	79	415	35
9	6396	45	6396	17	411	24	6396	38	411	29	6396	80	411	45
10	7360	43	7091	21	412	19	7094	87	412	35	7094	50	412	47

## STYLE 1, VARIATION 1(10X10X10, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	24046	461	17250	112	21319	280	17250	631	21319	130	17250	914	21319	124	17250	58
2	24980	426	20981	296	23953	249	20981	422	23953	169	20981	553	23953	183	20981	121
3	16953	419	15316	145	16775	263	15316	533	16775	173	15316	517	16775	144	15316	56
4	19025	552	14177	243	16074	146	14177	584	16074	161	14177	1060	16074	173	14177	77
5	18025	506	10347	132	13703	168	10347	458	13703	215	10347	959	13703	272	10347	75
6	18023	597	10347	128	13701	191	10347	558	13701	313	10347	241	13701	220	10347	88
7	24046	461	17250	112	21319	280	17250	631	21319	130	17250	914	21319	124	17250	58
8	21445	601	16828	231	21124	351	16828	388	21124	183	16828	728	21124	353	16828	70
9	25777	495	17704	236	20844	140	17704	461	20844	195	17704	607	20844	143	17704	66
10	23089	547	17466	333	20362	267	17466	549	20362	229	17466	622	20362	227	17466	76
<b><math>\Sigma_{si}=1.5</math></b>																
1	19994	565	14575	152	17804	236	14575	381	17804	208	14575	772	17804	111	14575	73
2	19579	711	13641	111	15032	175	13641	329	15033	173	13641	881	15033	102	13641	47
3	22381	777	11706	116	17811	331	11706	436	17811	132	11706	712	17811	181	11706	65
4	24897	585	20079	399	22154	125	20079	504	22154	119	20079	918	22154	77	20079	177
5	22829	604	14155	79	18808	229	14155	466	18808	187	14155	352	18802	159	14155	43
6	18352	398	12016	310	15849	128	12016	452	15849	177	12016	734	15849	193	12016	78
7	22595	342	14095	123	20396	272	14095	298	20396	117	14095	804	20396	146	14095	81
8	22381	777	11706	116	17811	331	11706	436	17811	132	11706	712	17811	181	11706	65
9	18003	581	14780	276	16008	151	14780	502	16008	227	14780	682	16008	215	14780	122
10	20123	467	17916	136	19899	239	17916	387	19899	111	17916	445	19899	113	17916	55
<b><math>\Sigma_{si}=2.0</math></b>																
1	20682	365	13734	165	16102	253	13734	281	16102	120	13734	188	16102	145	13734	230
2	19833	421	14594	231	17789	180	14594	349	17790	104	14594	290	17790	112	14594	73
3	24196	306	18592	352	20795	330	18592	332	20795	117	18592	657	20795	186	18592	90
4	20486	384	18369	144	20279	211	18369	290	20279	166	18369	295	20279	138	18369	46
5	16209	382	9401	177	12223	237	9401	287	12223	135	9401	310	12223	200	9401	49
6	24113	505	18434	340	23163	392	18434	338	23163	157	18434	328	23163	136	18434	108
7	17392	336	13560	172	16591	181	13560	454	16591	120	13560	768	16591	149	13560	88
8	22407	498	15975	136	18483	149	15975	402	18483	160	15975	398	18483	157	15975	66
9	15010	314	12325	152	14828	247	12325	496	14828	148	12325	204	14828	77	12325	84
10	19833	421	14594	231	17789	180	14594	349	17790	104	14594	290	17790	112	14594	73
<b><math>\Sigma_{si}=5.0</math></b>																
1	20341	258	13100	124	17599	113	13100	258	17599	66	13100	633	17599	134	13100	51
2	22463	262	15746	424	16993	114	15746	365	16993	200	15746	282	16993	91	15746	129
3	13721	123	11720	177	13721	188	11720	214	13721	73	11720	219	13721	114	11720	55
4	22843	307	11514	169	20250	226	11514	175	20250	89	11514	157	20250	83	11514	49
5	19776	361	11089	132	16354	166	11089	256	16354	151	11089	197	16354	94	11089	63
6	22061	258	13906	307	18461	166	13906	342	18461	92	13906	207	18461	120	13906	68
7	22840	387	11178	129	17108	258	11178	302	17109	74	11178	221	17109	62	11178	60
8	19731	372	14959	117	16769	169	14959	307	16769	99	14959	594	16769	71	14959	58
9	22843	307	11514	169	20250	226	11514	175	20250	89	11514	157	20250	83	11514	49
10	18331	316	12233	284	16765	208	12233	346	16765	117	12233	252	16765	145	12233	93



## STYLE 2, VARIATION 1(10X10X10, $\Sigma_{cap}=2.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	24028	144	17225	65	21301	89	17225	144	21301	167	17236	207	21301	139	17225	51
2	24980	255	20951	43	23939	80	20951	117	23939	123	20970	141	23939	221	20951	29
3	16942	107	15298	50	16764	84	15298	140	16764	202	15311	153	16764	195	15298	32
4	19013	207	14157	38	16062	97	14157	171	16062	172	14162	246	16062	158	14157	35
5	18014	278	10329	42	13690	118	10329	121	13690	225	10335	228	13690	179	10329	36
6	18011	301	10328	41	13688	82	10328	141	13688	200	10328	168	13688	201	10328	36
7	24028	144	17225	65	21301	89	17225	144	21301	167	17236	207	21301	139	17225	51
8	21433	187	16811	48	21112	92	16811	135	21112	136	16815	151	21112	156	16811	37
9	25768	179	17689	66	20836	117	17689	116	20836	134	17694	212	20836	222	17689	57
10	23081	144	17448	38	20355	100	17448	185	20355	172	17453	239	20355	180	17448	55
<b><math>\Sigma_{si}=1.5</math></b>																
1	19977	211	14552	46	17789	129	14552	149	17789	151	14564	208	17789	165	14552	52
2	19570	214	13623	37	15024	81	13623	124	15024	103	13632	228	15024	249	13623	34
3	22373	353	11691	41	17803	141	11691	214	17803	115	11701	157	17803	203	11691	51
4	24890	239	20066	36	22149	110	20066	247	22149	206	20073	185	22149	153	20066	40
5	22823	211	14150	57	18803	116	14150	107	18803	118	14152	221	18803	242	14150	54
6	18350	231	12006	25	15848	115	12006	73	15848	204	12008	115	15848	167	12006	19
7	22583	201	14071	26	20388	72	14071	83	20388	195	14085	333	20388	182	14071	20
8	22373	353	11691	41	17803	141	11691	214	17803	115	11701	157	17803	203	11691	51
9	17998	251	14772	52	16003	121	14772	122	16003	125	14778	234	16003	212	14772	47
10	20122	267	17909	54	19898	92	17909	128	19898	178	17910	119	19898	157	17909	47
<b><math>\Sigma_{si}=2.0</math></b>																
1	20678	267	13719	28	16097	54	13719	81	16097	211	13724	110	16097	209	13719	24
2	19826	141	14583	43	17784	79	14583	193	17784	169	14590	127	17784	166	14583	28
3	24195	171	18590	55	20795	145	18590	64	20795	176	18590	83	20795	214	18590	103
4	20483	119	18355	38	20276	88	18355	96	20276	172	18358	131	20276	158	18355	35
5	16203	111	9386	45	12215	120	9386	130	12215	157	9399	107	12215	191	9386	34
6	24109	117	18417	24	23160	64	18417	183	23160	222	18426	100	23160	184	18417	23
7	17382	96	13546	54	16581	104	13546	123	16581	182	13557	140	16581	180	13546	34
8	22404	244	15967	46	18482	104	15967	116	18482	169	15970	107	18482	187	15967	20
9	15006	117	12315	41	14825	70	12315	168	14825	140	12318	197	14825	145	12315	38
10	19826	141	14583	43	17784	79	14583	193	17784	169	14590	127	17784	166	14583	28
<b><math>\Sigma_{si}=5.0</math></b>																
1	20334	135	13084	48	17592	125	13084	126	17592	183	13095	166	17592	159	13084	42
2	22463	197	15741	30	16991	100	15741	111	16991	103	15746	161	16991	260	15741	30
3	13715	92	11708	41	13715	70	11708	144	13715	190	11719	155	13715	147	11708	42
4	22843	184	11512	38	20250	77	11512	116	20250	171	11512	175	20250	180	11512	40
5	19773	210	11081	44	16350	79	11081	153	16350	161	11088	197	16350	159	11081	41
6	22060	162	13906	26	18460	126	13906	110	18460	151	13906	111	18460	151	13906	24
7	22839	286	11173	33	17106	74	11173	150	17106	199	11175	205	17106	233	11173	33
8	19731	161	14952	46	16768	74	14952	143	16768	123	14952	202	16768	171	14952	47
9	22843	184	11512	38	20250	77	11512	116	20250	171	11512	175	20250	180	11512	40
10	18331	150	12233	38	16765	87	12233	120	16765	131	12233	225	16765	241	12233	34

$\Sigma si=10.0$ 

1	12962	135	11437	55	12962	45	11437	117	12962	137	11437	79	12962	272	11437	47
2	25750	298	19145	47	23131	99	19145	88	23131	225	19145	107	23131	228	19145	34
3	15496	145	9880	58	15066	93	9880	73	15066	162	9880	109	15066	159	9880	102
4	12302	157	10881	53	11007	69	10881	114	11007	174	10881	85	11007	178	10881	52
5	22451	166	17041	49	18809	77	17041	146	18809	181	17041	151	18809	155	17041	42
6	15065	113	10111	91	12964	116	10111	77	12964	154	10111	83	12964	204	10111	57
7	22597	214	13059	38	18815	112	13059	115	18815	204	13059	186	18815	199	13059	38
8	19159	176	13913	36	16762	99	13913	130	16762	120	13913	196	16762	198	13913	32
9	21842	157	15042	28	21314	108	15042	110	21314	130	15042	91	21314	227	15042	34
10	12302	157	10881	53	11007	69	10881	114	11007	174	10881	85	11007	178	10881	52

 $\Sigma si=20.0$ 

1	18933	162	13122	23	14766	92	13122	102	14766	202	13122	232	14766	331	13122	18
2	22794	117	17874	28	22168	78	17874	138	22168	172	17874	109	22168	260	17874	19
3	23262	197	18317	69	19524	100	18317	118	19524	191	18317	187	19524	348	18317	44
4	18217	210	15687	30	17578	96	15687	102	17578	198	15687	172	17578	328	15687	20
5	21225	214	16642	48	18077	73	16642	128	18077	124	16642	216	18077	129	16642	46
6	18137	263	12493	26	14965	86	12493	113	14965	159	12493	97	14965	197	12493	24
7	20466	179	12441	41	17740	81	12441	141	17740	182	12442	132	17440	238	12241	37
8	18933	162	13122	23	14766	92	13122	102	14766	202	13122	232	14766	331	13122	18
9	20427	250	15763	59	17566	106	15763	123	17566	156	15771	137	17569	156	15763	51
10	21360	212	15556	34	17651	79	15556	119	17651	155	15556	170	17651	156	15556	34

 $\Sigma si=30.0$ 

1	25114	257	15672	24	19437	94	15672	139	19437	135	15672	121	19437	179	15672	28
2	22848	256	15286	35	18290	83	15286	100	18290	140	15286	201	18290	184	15286	50
3	23524	210	14777	47	20270	106	14777	131	20270	111	14777	186	20270	156	14777	35
4	20207	142	13430	32	18084	87	13430	158	18084	136	13430	208	18084	237	13430	32
5	20526	139	15183	28	16114	123	15183	183	16114	207	15183	114	16114	205	15183	23
6	13792	94	12581	55	13791	112	12581	142	13791	210	12583	246	13791	152	12581	37
7	12194	131	9838	37	12194	84	9838	154	12194	177	9840	181	12194	245	9838	36
8	23524	210	14777	47	20270	106	14777	131	20270	111	14777	186	20270	156	14777	35
9	21225	214	16642	48	18077	73	16642	128	18077	124	16642	216	18077	129	16642	46
10	12989	151	7664	24	10881	59	7664	79	10881	165	7664	111	10881	218	7664	17

 $\Sigma si=40.0$ 

1	22025	200	14999	40	19311	79	14999	162	19312	177	14999	148	19312	175	14999	36
2	21568	183	13521	27	17104	121	13521	90	17104	107	13521	84	17104	215	13521	25
3	22271	203	12307	26	18080	63	12307	165	18080	121	12311	97	18080	176	12307	22
4	20720	142	12368	36	19830	70	12368	162	19830	98	12368	170	19830	206	12368	36
5	27982	258	21448	51	23280	81	21448	98	23280	186	21448	214	23280	175	21448	36
6	18160	133	14107	26	16140	89	14107	93	16140	180	14107	273	16140	243	14107	26
7	20376	146	15642	50	19695	99	15642	117	19695	133	15643	224	19695	160	15642	38
8	17973	118	9929	46	12246	113	9929	125	12246	108	9929	185	12246	155	9929	32
9	12164	127	11335	41	12040	114	11335	152	12040	181	11335	123	12040	166	11335	34
10	20720	142	12368	36	19830	70	12368	162	19830	98	12368	170	19830	206	12368	36

 $\Sigma si=50.0$ 

1	22213	153	18421	42	20324	103	18421	176	20324	214	18421	142	20324	148	18421	42
2	16323	132	12767	31	14870	85	12767	115	14871	135	12767	275	14871	215	12767	26
3	24657	364	19264	43	19813	104	19264	122	19813	202	19278	196	19813	239	19264	37
4	12265	153	9943	46	11726	77	9943	117	11726	143	9943	243	11726	198	9943	34
5	20474	257	12505	42	16452	106	12505	90	16452	141	12505	167	16452	152	12505	42
6	22190	179	16202	26	18856	71	16202	126	18856	148	16202	124	18856	236	16202	20
7	19192	157	13038	42	14615	102	13038	112	14615	152	13038	168	14615	213	13038	42
8	19407	99	14524	33	16679	65	14524	104	16680	133	14524	137	16680	211	14524	31
9	16323	132	12767	31	14870	85	12767	115	14871	135	12767	275	14871	215	12767	26
10	21339	186	16944	42	19960	70	16944	116	19660	135	16944	201	19960	153	16944	31

## STYLE 1, VARIATION 2(10X10X10, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	24046	1610	7620	203	414	146	7628	374	414	398	7639	508	414	330
2	24980	705	8330	373	415	122	8341	467	415	329	8354	410	415	503
3	16953	808	7373	321	420	132	7382	879	420	282	7389	429	420	304
4	19025	1164	6298	296	414	197	6308	572	414	559	6317	982	414	400
5	18025	1255	5787	174	414	153	5793	493	414	368	5805	942	414	328
6	18023	1345	5786	304	414	132	5792	708	414	398	5803	1109	414	382
7	24046	1610	7620	203	414	146	7628	374	414	398	7639	508	414	330
8	21445	1706	6856	231	409	112	6870	1142	409	453	6885	962	409	288
9	25777	1518	8148	192	417	126	8159	362	417	365	8168	965	417	342
10	23089	1232	7514	251	415	353	7527	697	415	210	7539	1220	415	857
<b><math>\Sigma_{si}=1.5</math></b>														
1	19994	1005	7020	270	413	178	7032	427	413	394	7039	832	413	422
2	19579	1110	6851	262	410	283	6864	1059	410	430	6873	757	410	325
3	22381	1149	7569	208	412	107	7580	733	412	466	7587	704	412	405
4	24897	1416	7916	152	409	112	7927	765	409	327	7937	734	409	232
5	22829	1272	7794	258	414	116	7811	450	414	248	7819	781	414	261
6	18352	907	6234	211	407	107	6250	660	407	352	6257	957	407	291
7	22595	1339	7435	236	409	150	7446	649	409	402	7457	801	409	281
8	22381	1149	7569	208	412	107	7580	733	412	466	7587	704	412	405
9	18003	933	6124	298	414	114	6136	962	414	620	6147	458	414	514
10	20123	1110	6459	178	412	164	6471	320	412	326	6480	710	412	331
<b><math>\Sigma_{si}=2.0</math></b>														
1	20682	859	7218	135	412	246	7231	692	412	192	7241	716	412	252
2	19833	909	8126	257	414	126	8141	346	414	190	8150	392	414	350
3	24196	1051	7716	211	409	147	7723	432	409	201	7728	764	409	277
4	20486	1247	6294	396	412	111	6307	568	412	325	6315	615	412	339
5	16209	813	5696	148	404	72	5704	391	404	334	5709	563	404	352
6	24113	1585	6605	281	417	212	6617	681	416	393	6624	705	416	273
7	17392	435	8115	291	411	154	8125	364	411	311	8129	791	411	301
8	22407	1048	7548	189	412	180	7561	428	412	220	7568	709	412	256
9	15010	726	6945	178	410	143	6955	666	410	210	6963	516	410	304
10	19833	909	8126	257	414	126	8141	346	414	190	8150	392	414	350
<b><math>\Sigma_{si}=5.0</math></b>														
1	20341	804	7140	280	411	151	7153	632	411	324	7156	516	411	301
2	22463	711	7123	113	417	91	7130	348	417	211	7132	514	417	223
3	13721	434	6752	82	411	101	6760	238	411	237	6762	444	411	292
4	22843	474	7431	175	407	150	7438	210	407	329	7440	226	407	245
5	19776	662	6733	85	410	90	6742	265	410	150	6745	568	410	194
6	22061	558	7469	231	408	70	7477	246	408	331	7479	575	408	238
7	22840	661	7200	149	412	89	7208	300	412	167	7211	395	412	204
8	19731	708	6257	114	412	180	6270	550	412	615	6271	233	412	170
9	22843	474	7431	175	407	150	7438	210	407	329	7440	226	407	245
10	18331	657	6126	132	404	155	6142	258	404	176	6146	260	404	230

**$\Sigma si=10.0$** 

1	12962	334	6428	134	405	79	6442	251	405	303	6446	514	405	166
2	25750	1272	6931	132	404	63	6946	273	404	302	6949	231	404	297
3	15498	327	6458	99	412	69	6467	228	412	215	6467	238	412	155
4	12302	354	6290	74	407	67	6301	247	407	242	6303	232	407	228
5	22451	861	7413	120	416	129	7424	261	416	260	7426	325	416	402
6	15065	388	6006	132	419	83	6013	303	419	221	6013	229	419	188
7	22599	698	7560	153	407	91	7575	315	407	343	7576	666	407	210
8	19159	512	6757	87	409	74	6769	240	409	364	6770	200	409	201
9	21842	698	6754	113	413	71	6763	289	413	220	6765	499	413	168
10	12302	354	6290	74	407	67	6301	247	407	242	6303	232	407	228

 **$\Sigma si=20.0$** 

1	18933	530	6259	131	408	61	6270	513	408	158	6271	215	408	173
2	22794	678	7665	163	407	69	7681	272	407	286	7681	510	407	174
3	23262	860	6895	89	410	90	6910	239	410	121	6910	216	410	184
4	18217	582	6312	136	412	64	6325	244	412	151	6325	390	412	246
5	21227	668	6737	151	408	67	6747	263	408	229	6747	239	408	177
6	18137	603	6346	113	406	113	6358	299	406	131	6360	507	406	261
7	20467	623	6843	169	409	62	6856	248	409	315	6856	459	409	230
8	18933	530	6259	131	408	61	6270	513	408	158	6271	215	408	173
9	20427	697	6572	88	409	84	6586	295	409	160	6587	246	409	193
10	21360	654	7291	132	412	66	7299	248	412	263	7299	239	412	190

 **$\Sigma si=30.0$** 

1	25114	576	8444	206	409	66	8456	238	409	306	8456	177	409	430
2	22848	741	7304	131	408	76	7318	623	408	237	7320	227	408	204
3	23524	986	6093	157	405	58	6106	221	405	141	6106	413	405	240
4	20207	507	7022	102	412	74	7034	378	412	243	7034	236	412	124
5	20526	508	7200	80	408	73	7210	250	408	350	7210	206	408	169
6	13792	372	6999	256	406	101	7009	570	406	318	7011	493	406	204
7	12195	359	6001	112	413	160	6010	268	413	190	6010	421	413	265
8	23524	986	6093	157	405	58	6106	221	405	141	6106	413	405	240
9	21227	668	6737	151	408	67	6747	263	408	229	6747	239	408	177
10	12989	341	5982	138	407	77	5989	285	407	214	5989	354	407	167

 **$\Sigma si=40.0$** 

1	22025	556	7410	139	406	140	7424	455	406	271	7424	189	406	241
2	21568	539	6814	189	411	81	6824	252	411	332	6824	237	411	197
3	22274	529	7297	130	411	90	7307	286	411	167	7308	217	411	196
4	20720	661	6873	65	408	72	6884	308	408	262	6884	298	408	187
5	27982	1130	7016	184	412	132	7028	237	412	521	7028	259	412	160
6	18160	588	5939	104	413	73	5949	226	413	82	5949	228	413	168
7	20376	626	6683	102	410	68	6691	232	410	351	6691	243	410	244
8	17973	544	6257	124	406	67	6273	238	406	253	6273	397	406	141
9	12164	321	6118	97	412	85	6129	402	412	156	6129	415	412	239
10	20720	661	6873	65	408	72	6884	308	408	262	6884	298	408	187

 **$\Sigma si=50.0$** 

1	22213	784	6405	125	405	57	6419	238	405	152	6419	211	405	220
2	16323	319	7229	98	411	88	7242	225	411	304	7242	388	411	172
3	24665	748	7525	127	410	79	7536	239	410	194	7536	228	410	233
4	12265	266	6223	96	407	69	6235	279	407	326	6235	505	407	169
5	20474	570	6682	106	412	108	6691	238	412	222	6691	162	412	235
6	22190	574	7106	81	408	63	7116	265	408	204	7116	196	408	178
7	19192	442	6721	88	409	67	6727	222	409	141	6727	251	409	191
8	19407	463	6358	90	410	71	6372	248	410	208	6372	208	410	229
9	16323	319	7229	98	411	88	7242	225	411	304	7242	388	411	172
10	21339	684	6604	109	414	65	6618	226	414	283	6618	217	414	236

## STYLE 2, VARIATION 2(10X10X10, $\Sigma_{cap}=2.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	24028	557	7607	49	408	81	7614	127	408	94	7639	306	408	139
2	24980	871	8318	76	409	72	8330	137	409	110	8354	717	409	104
3	16942	305	7361	67	414	69	7370	150	414	136	7389	730	414	123
4	19013	574	6286	66	405	81	6297	148	405	139	6317	670	405	121
5	18014	677	5779	74	407	67	5785	161	407	126	5805	830	407	182
6	18011	578	5776	73	407	65	5779	168	407	122	5791	624	407	123
7	24028	557	7607	49	408	81	7614	127	408	94	7639	306	408	139
8	21433	633	6852	81	406	62	6866	137	406	132	6885	540	406	128
9	25768	738	8141	81	411	73	8152	167	411	125	8168	700	411	147
10	23081	608	7510	66	412	77	7523	160	412	173	7539	675	412	139
<b><math>\Sigma_{si}=1.5</math></b>														
1	19977	513	7006	66	408	83	7018	155	408	189	7039	265	408	138
2	19570	581	6848	54	407	60	6862	130	407	76	6873	589	407	123
3	22373	677	7562	71	410	84	7573	144	410	161	7587	299	410	139
4	24890	848	7913	64	408	96	7924	121	408	150	7937	194	408	147
5	22823	788	7787	59	409	69	7803	135	409	148	7819	564	409	111
6	18350	613	6231	56	406	60	6247	153	406	139	6257	476	406	125
7	22583	1053	7427	60	407	74	7438	140	407	117	7457	566	407	146
8	22373	677	7562	71	410	84	7573	144	410	161	7587	299	410	139
9	17998	532	6120	63	412	59	6132	153	412	112	6147	202	412	125
10	20122	662	6458	91	412	82	6470	134	412	147	6480	617	412	191
<b><math>\Sigma_{si}=2.0</math></b>														
1	20678	617	7215	61	410	63	7228	161	410	112	7241	174	410	152
2	19826	577	8121	79	408	87	8136	180	408	103	8150	533	408	119
3	24195	872	7715	64	408	90	7722	132	408	146	7728	209	408	114
4	20483	835	6290	54	409	63	6303	189	409	70	6315	210	409	116
5	16203	438	5689	55	404	91	5696	148	404	124	5709	456	404	115
6	24109	1350	6601	75	414	72	6613	136	414	101	6624	537	414	190
7	17382	205	8107	66	407	83	8117	130	407	92	8129	194	407	138
8	22404	664	7542	77	409	71	7556	124	409	99	7568	406	409	126
9	15006	355	6942	86	408	78	6952	137	408	169	6963	187	408	169
10	19826	577	8121	79	408	87	8136	180	408	103	8150	533	408	119
<b><math>\Sigma_{si}=5.0</math></b>														
1	20334	524	7133	89	410	84	7146	165	410	164	7156	429	410	177
2	22463	658	7121	70	416	82	7129	159	416	114	7132	370	416	110
3	13715	251	6750	68	411	75	6758	136	411	153	6762	168	411	181
4	22843	445	7431	50	407	87	7438	125	407	157	7440	174	407	176
5	19773	585	6729	74	409	84	6738	152	409	143	6745	150	409	125
6	22060	567	7468	77	407	85	7476	153	407	122	7479	439	407	150
7	22839	578	7199	79	412	73	7207	165	411	94	7211	318	412	125
8	19731	655	6257	46	412	85	6269	152	412	104	6271	182	412	136
9	22843	445	7431	50	407	87	7438	125	407	157	7440	174	407	176
10	18331	550	6126	53	404	62	6142	130	404	84	6146	193	404	107

$\Sigma si=10.0$ 

1	12962	300	6428	68	405	70	6442	121	405	130	6446	168	405	176
2	25750	1596	6930	81	404	56	6944	130	404	94	6949	137	404	104
3	15496	432	6458	64	412	75	6467	127	412	95	6467	141	412	135
4	12302	225	6290	58	407	79	6301	121	407	123	6303	157	407	108
5	22451	726	7412	71	416	73	7423	154	416	173	7426	191	416	124
6	15065	477	6006	97	419	84	6013	133	419	131	6013	376	419	175
7	22597	545	7560	68	407	83	7575	165	407	142	7576	171	407	172
8	19159	585	6757	70	409	63	6769	145	409	146	6770	133	409	120
9	21842	751	6754	49	413	63	6763	152	413	80	6765	342	413	112
10	12302	225	6290	58	407	79	6301	121	407	123	6303	157	407	108

 $\Sigma si=20.0$ 

1	18933	657	6259	48	408	80	6270	151	408	128	6271	144	408	153
2	22794	847	7665	58	407	57	7681	137	407	103	7681	184	407	107
3	23262	1210	6895	100	410	65	6910	131	410	81	6910	177	410	143
4	18217	868	6312	62	412	64	6325	152	412	121	6325	165	412	108
5	21225	809	6737	68	407	52	6747	158	407	63	6747	141	407	87
6	18137	633	6346	73	406	75	6358	125	406	156	6360	141	406	176
7	20466	623	6843	82	409	78	6856	139	409	156	6856	138	409	175
8	18933	657	6259	48	408	80	6270	151	408	128	6271	144	408	153
9	20427	888	6570	117	409	78	6583	193	409	166	6587	183	409	146
10	21360	515	7291	67	412	67	7299	158	412	92	7299	151	412	99

 $\Sigma si=30.0$ 

1	25114	551	8442	67	409	85	8454	154	409	125	8456	132	409	144
2	22848	717	7304	66	408	53	7318	133	408	155	7320	132	408	123
3	23524	1006	6093	58	405	75	6106	138	405	159	6106	135	405	121
4	20207	594	7022	74	412	80	7034	150	412	132	7034	155	412	147
5	20526	501	7200	75	408	73	7210	128	408	191	7210	142	408	154
6	13792	248	6998	86	406	68	7008	128	406	166	7011	145	406	155
7	12195	300	6001	88	413	56	6010	138	413	162	6010	135	413	155
8	23524	1006	6093	58	405	75	6106	138	405	159	6106	135	405	121
9	21225	809	6737	68	407	52	6747	158	407	63	6747	141	407	87
10	12989	395	5982	41	407	49	5989	137	407	125	5989	121	407	143

 $\Sigma si=40.0$ 

1	22025	667	7410	87	406	66	7424	153	406	141	7424	153	406	130
2	21568	611	6814	62	411	68	6824	148	411	119	6824	176	411	104
3	22271	673	7295	73	411	79	7305	148	411	102	7308	145	411	137
4	20720	668	6873	74	408	79	6884	129	408	92	6884	200	408	167
5	27982	1486	7016	82	412	67	7028	145	412	132	7028	150	412	147
6	18160	603	5939	74	413	70	5949	113	413	127	5949	144	413	158
7	20376	721	6682	73	410	73	6690	132	410	144	6691	154	410	143
8	17973	576	6257	51	406	78	6273	151	406	107	6273	141	406	108
9	12164	227	6118	64	412	58	6129	133	412	131	6129	128	412	154
10	20720	668	6873	74	408	79	6884	129	408	92	6884	200	408	167

 $\Sigma si=50.0$ 

1	22213	1082	6405	73	405	70	6419	124	405	143	6419	140	405	116
2	16323	359	7229	79	411	74	7242	143	411	129	7242	125	411	173
3	24657	757	7525	66	407	74	7536	157	407	164	7536	171	407	202
4	12265	259	6223	63	407	74	6235	137	407	93	6235	164	407	119
5	20474	564	6682	81	412	75	6691	121	412	111	6691	135	412	127
6	22190	581	7106	63	408	70	7116	156	408	111	7116	152	408	111
7	19192	423	6721	62	409	75	6727	156	409	122	6727	136	409	119
8	19407	577	6358	52	410	77	6372	128	410	79	6372	135	410	99
9	16323	359	7229	79	411	74	7242	143	411	129	7242	125	411	173
10	21339	737	6604	56	414	66	6618	145	414	104	6618	157	414	105



# STYLE 1, VARIATION 1(10X10X10, $\Sigma_{cap}=6.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER

## $\Sigma_{si}=1.0$

1	9134	415	9134	244	6361	121	9134	559	6361	150	9134	1573	6361	184	6361	47
2	10103	304	8823	295	5442	271	8823	628	5442	117	8823	520	5442	120	5442	51
3	6601	439	6601	139	3122	199	6601	314	3122	91	6601	497	3122	88	3122	49
4	9079	700	5972	280	5990	148	5972	802	5990	150	5972	823	5990	168	5972	83
5	5620	316	5620	152	4463	159	5620	568	4463	152	5620	622	4463	113	4463	82
6	5621	366	5621	147	4462	138	5621	551	4462	134	5621	1304	4462	104	4462	95
7	9136	412	9136	244	6362	120	9136	559	6362	162	9136	1102	6362	182	6362	42
8	7363	449	6657	142	6698	202	6657	1373	6698	152	6657	1287	6698	128	6657	65
9	7517	230	7517	96	6544	335	7517	431	6544	93	7517	561	6544	102	6544	66
10	6864	320	6864	141	3962	166	6864	624	3962	199	6864	327	3962	177	3962	55

## $\Sigma_{si}=1.5$

1	8075	537	5771	253	4987	156	5771	524	4987	172	5771	629	4987	102	4987	98
2	8756	247	8756	140	4577	109	8756	519	4577	89	8756	506	4577	88	4577	52
3	7362	364	7362	69	3291	214	7362	411	3291	101	7362	914	3291	75	3291	73
4	7461	364	7461	149	4824	164	7461	854	4824	82	7461	1127	4824	77	4824	227
5	5858	261	5858	151	4687	236	5858	496	4687	92	5858	1129	4687	105	4687	68
6	6845	380	6840	150	3429	109	6845	482	3429	93	6845	980	3429	96	3429	63
7	9660	462	7499	241	3921	93	7499	675	3921	95	7499	352	3921	101	3921	72
8	7360	364	7360	69	3291	214	7360	411	3291	101	7360	914	3291	75	3291	73
9	6898	297	6898	160	4243	220	6898	565	4243	104	6898	633	4243	94	4243	100
10	7247	401	7247	89	3924	166	7247	292	3924	134	7247	739	3924	105	3924	59

## $\Sigma_{si}=2.0$

1	6713	441	6713	145	2769	201	6713	604	2769	139	6713	496	2769	155	2769	74
2	7460	350	7460	120	3796	141	7460	714	3796	101	7460	710	3796	102	3796	73
3	7800	245	7800	155	3521	182	7800	803	3521	75	7800	441	3521	60	3521	87
4	8652	529	7238	213	5726	104	7238	560	5726	115	7238	650	5726	104	5726	60
5	5640	251	5640	104	3082	149	5640	456	3082	133	5640	392	3082	120	3082	38
6	5928	290	5928	97	3503	232	5928	676	3503	139	5928	921	3503	84	3503	86
7	9358	450	7294	269	7450	144	7294	407	7450	118	7294	519	7450	155	7294	83
8	9038	314	9038	118	4010	228	9038	540	4010	115	9038	454	4010	98	4010	30
9	7480	292	7480	265	3218	162	7480	433	3218	104	7480	700	3218	83	3218	40
10	7460	350	7460	120	3796	141	7460	714	3796	101	7460	710	3796	102	3796	73

## $\Sigma_{si}=5.0$

1	7138	265	7138	61	4581	160	7138	318	4581	101	7138	296	4581	115	4581	67
2	8009	391	8009	157	6181	100	8009	368	6181	70	8009	367	6181	65	6181	56
3	7804	184	7804	156	4087	131	7804	441	4087	77	7804	490	4087	76	4087	58
4	5528	304	5528	67	3500	133	5528	341	3500	57	5528	336	3500	55	3500	47
5	6506	266	6506	118	2714	88	6506	382	2714	55	6506	245	2714	54	2714	55
6	7589	272	7589	82	4824	125	7589	363	4824	67	7589	326	4824	59	4824	62
7	9409	237	9409	111	3552	138	9409	385	3552	54	9409	360	3552	58	3552	48
8	5604	253	5604	75	4179	91	5604	319	4179	101	5604	345	4179	114	4179	58
9	5530	304	5530	67	3500	133	5530	341	3500	57	5530	336	3500	55	3500	47
10	6987	279	6987	137	3721	111	6987	659	3721	87	6987	437	3721	75	3721	59

$\Sigma si=10.0$ 

1	7040	269	7040	69	2872	166	7040	367	2872	51	7040	390	2872	53	2872	98
2	6885	247	6885	164	4181	147	6885	433	4181	54	6885	328	4181	54	4181	46
3	10120	262	6073	214	6083	76	6073	406	6083	71	6073	293	6083	57	6073	113
4	8590	189	6314	199	3609	127	6314	389	3609	96	6314	212	3609	55	3609	106
5	8786	192	8786	92	6080	158	8786	306	6080	68	8786	241	6080	72	6080	30
6	7657	208	6269	177	4629	141	6269	350	4629	48	6269	176	4629	54	4629	106
7	9898	230	8589	165	4710	136	8589	500	4710	61	8589	392	4710	66	4710	99
8	6403	162	6403	86	2948	109	6403	188	2948	52	6403	274	2948	67	2948	92
9	6534	146	6534	95	2880	144	6534	296	2880	69	6534	292	2880	48	2880	48
10	8592	191	6314	168	3609	128	6314	422	3609	98	6314	212	3609	56	3609	106

 $\Sigma si=20.0$ 

1	7967	225	7967	58	4020	99	7967	181	4020	57	7967	272	4020	59	4020	102
2	7471	106	7471	130	2810	181	7471	236	2810	95	7471	87	2810	50	2810	96
3	6332	191	6332	52	3166	76	6332	294	3166	82	6332	196	3166	53	3166	108
4	6088	243	6088	105	3484	140	6088	413	3484	52	6088	231	3484	48	3484	60
5	5940	172	5940	72	4512	133	5940	437	4512	48	5940	314	4512	49	4512	95
6	8318	183	7296	195	4011	133	7296	401	4011	58	7296	424	4011	63	4011	47
7	5806	193	5806	76	5272	78	5806	414	5272	123	5806	255	5272	118	5272	109
8	7969	228	7968	58	4020	101	7968	180	4020	59	7968	202	4020	59	4020	102
9	6587	128	6587	124	3059	103	6587	325	3059	39	6587	421	3059	52	3059	57
10	5860	248	5860	125	5552	110	5860	361	5552	59	5860	312	5552	56	5552	104

 $\Sigma si=30.0$ 

1	9668	133	9668	251	4430	99	9668	316	4430	67	9668	286	4430	94	4430	101
2	6372	241	6372	152	4120	109	6372	267	4120	64	6372	250	4120	53	4120	53
3	7149	118	7149	124	3215	109	7149	306	3215	52	7149	204	3215	72	3215	82
4	8097	207	6909	161	6840	88	6909	403	6840	47	6909	220	6840	51	6840	91
5	8932	201	8932	313	4582	143	8932	329	4582	89	8932	248	4582	119	4582	56
6	6284	211	6284	156	5309	166	6284	413	5309	124	6284	749	5309	110	5309	57
7	6137	294	6137	54	5307	269	6137	564	5307	154	6137	406	5307	183	5307	104
8	7148	121	7148	122	3215	111	7148	333	3215	52	7148	215	3215	76	3215	88
9	5940	172	5940	72	4512	133	5940	437	4512	48	5940	314	4512	49	4512	95
10	7710	228	7710	265	5485	117	7710	365	5485	107	7710	321	5485	97	5485	54

 $\Sigma si=40.0$ 

1	9460	207	7720	260	4854	194	7720	315	4854	80	7720	324	4854	98	4854	76
2	7737	184	7737	194	6420	250	7737	408	6420	120	7737	299	6420	124	6420	59
3	6559	199	6559	357	2838	142	6559	293	2838	103	6559	296	2838	106	2838	55
4	7236	167	6653	292	5491	123	6653	515	5491	97	6653	416	5491	102	5491	72
5	8919	255	8919	295	5513	98	8919	387	5513	93	8919	275	5513	110	5513	99
6	6148	187	6148	104	3138	171	6148	378	3138	95	6148	445	3138	130	3138	101
7	6671	220	6671	270	3614	197	6671	270	3614	100	6671	407	3614	102	3614	112
8	6413	235	6413	106	5103	109	6413	260	5103	160	6413	184	5103	156	5103	52
9	6429	224	6002	177	4810	125	6002	375	4810	146	6002	421	4810	209	4810	47
10	7240	156	6656	232	5491	125	6656	515	5491	102	6656	416	5491	105	5491	77

 $\Sigma si=50.0$ 

1	9035	358	8334	20	7259	103	8334	182	7259	94	8334	148	7259	117	7259	29
2	5880	246	5880	70	3860	103	5880	432	3860	82	5880	306	3860	107	3860	64
3	5992	199	5992	52	4045	146	5992	746	4045	71	5992	348	4045	122	4045	35
4	5656	192	5656	109	4839	151	5656	328	4839	86	5656	387	4839	96	4839	72
5	5755	175	5755	70	4526	159	5755	501	4526	148	5755	320	4526	116	4526	103
6	8891	237	8222	45	6813	107	8222	192	6813	105	8222	377	6813	177	6813	35
7	6426	205	6426	67	4401	137	6426	458	4401	95	6426	362	4401	100	4401	80
8	9053	140	9053	105	5354	145	9053	410	5354	110	9053	342	5354	139	5354	43
9	7116	204	7116	100	3508	99	7116	357	3508	77	7116	256	3508	80	3508	98
10	5658	201	5658	111	4839	112	5658	323	4839	86	5658	387	4839	86	4839	77



## STYLE 2, VARIATION 1(10X10X10, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
$\Sigma_{si}=1.0$																
1	9119	110	9119	38	6344	61	9119	358	6344	112	9134	619	6344	143	6344	34
2	10096	155	8800	39	5425	125	8800	153	5425	77	8822	242	5425	138	5425	31
3	6583	107	6583	57	3104	97	6583	185	3104	105	6601	194	3104	158	3104	31
4	9053	251	5952	35	5972	127	5952	127	5972	79	5970	263	5972	80	5952	35
5	5603	102	5603	32	4444	74	5603	178	4444	131	5620	344	4444	162	4444	32
6	5603	102	5603	32	4443	103	5603	172	4443	100	5621	360	4443	148	4443	32
7	9116	112	9110	38	6344	63	9116	320	6344	111	9136	620	6344	152	6344	34
8	7349	158	6639	31	6685	141	6639	136	6685	124	6653	231	6685	146	6639	31
9	7501	95	7501	56	6528	117	7501	135	6528	125	7517	217	6528	117	6528	53
10	6847	86	6847	41	3950	147	6847	165	3950	124	6864	205	3950	193	3950	33
$\Sigma_{si}=1.5$																
1	8052	185	5751	44	4964	76	5751	213	4964	128	5771	201	4964	125	4964	49
2	8738	82	8738	38	4559	69	8738	248	4559	82	8756	231	4559	141	4559	38
3	7350	103	7350	71	3276	82	7350	126	3276	187	7362	159	3276	136	3276	58
4	7451	99	7451	61	4814	98	7451	327	4814	121	7461	573	4814	97	4814	50
5	5846	102	5846	43	4681	92	5846	172	4681	108	5858	336	4681	133	4681	34
6	6832	57	6832	48	3416	73	6832	361	3416	97	6845	338	3416	99	3416	48
7	9640	219	7481	54	3901	75	7481	171	3901	122	7497	218	3901	109	3901	54
8	7352	103	7352	71	3276	82	7352	126	3276	187	7352	159	3276	136	3276	58
9	6890	103	6890	51	4234	85	6890	114	4234	94	6898	307	4234	117	4234	59
10	7241	85	7241	53	3918	133	7241	271	3918	111	7247	153	3918	95	3918	47
$\Sigma_{si}=2.0$																
1	6694	89	6694	39	2750	75	6694	126	2750	121	6713	191	2750	119	2750	38
2	7445	93	7445	48	3787	120	7445	269	3787	124	7460	443	3787	137	3787	39
3	7785	84	7785	31	3505	94	7785	174	3505	129	7800	313	3505	161	3505	31
4	8638	238	7233	26	5720	73	7233	128	5720	128	7236	276	5720	151	5720	50
5	5625	52	5625	39	3067	71	5625	128	3067	132	5640	155	3067	119	3067	30
6	5923	99	5923	42	3498	110	5923	369	3498	114	5928	554	3498	128	3498	42
7	9348	244	7284	47	7442	98	7284	160	7442	131	7294	198	7442	158	7284	47
8	9031	88	9031	36	4000	106	9031	283	4000	142	9038	225	4000	168	4000	34
9	7476	88	7476	56	3214	96	7476	130	3214	93	7480	441	3214	102	3214	36
10	7445	93	7445	48	3787	120	7445	269	3787	124	7460	443	3787	137	3787	39
$\Sigma_{si}=5.0$																
1	7121	91	7121	42	4569	125	7121	302	4569	121	7138	362	4569	142	4569	61
2	8005	107	8005	42	6175	69	8005	305	6175	95	8009	444	6175	101	6175	31
3	7797	54	7797	29	4080	77	7797	367	4080	116	7804	169	4080	150	4080	30
4	5526	81	5526	61	3499	92	5526	136	3499	132	5528	188	3499	121	3499	55
5	6498	82	6498	36	2706	99	6498	160	2706	118	6506	333	2706	125	2706	36
6	7579	81	7579	40	4814	69	7579	148	4814	128	7589	274	4814	134	4814	34
7	9404	96	9404	34	3546	66	9404	121	3546	178	9409	156	3546	130	3546	35
8	5603	109	5603	45	4178	141	5603	135	4178	157	5604	146	4178	116	4178	35
9	5528	81	5528	61	3499	92	5528	136	3499	132	5530	188	3499	121	3499	55
10	6987	107	6987	37	3721	64	6987	151	3721	85	6987	230	3721	111	3721	33

$\Sigma si=10.0$ 

1	7040	119	7040	48	2872	92	7040	163	2872	79	7040	367	2872	134	2872	36
2	6877	114	6877	51	4179	69	6877	104	4179	100	6885	308	4179	113	4179	30
3	10113	299	6073	32	6083	86	6073	222	6083	104	6073	201	6083	123	6073	29
4	8590	245	6314	39	3609	65	6314	133	3609	113	6314	190	3609	165	3609	36
5	8779	108	8779	30	6074	103	8779	276	6074	130	8786	486	6074	126	6074	32
6	7657	178	6269	36	4629	98	6269	140	4629	87	6269	202	4629	137	4629	30
7	9898	185	8584	64	4710	96	8584	155	4710	100	8589	189	4710	151	4710	56
8	6403	110	6403	64	2948	72	6403	262	2948	112	6403	139	2948	189	2948	31
9	6531	108	6531	50	2877	90	6531	112	2877	82	6534	501	2880	117	2877	51
10	8592	245	6314	55	3609	88	6314	133	3609	113	6314	190	3609	113	3609	36

 $\Sigma si=20.0$ 

1	7967	114	7967	31	4019	107	7967	146	4019	104	7967	151	4019	157	4019	42
2	7471	81	7471	64	2810	129	7471	130	2810	181	7471	175	2810	152	2810	35
3	6332	94	6332	57	3166	90	6332	163	3166	119	6332	225	3166	168	3166	54
4	6088	72	6088	37	3484	126	6088	172	3484	105	6088	131	3484	106	3484	37
5	5940	98	5940	37	4512	105	5940	130	4512	110	5940	173	4512	101	4512	34
6	8316	148	7296	65	4009	111	7296	188	4009	103	7296	153	4009	140	4009	58
7	5806	100	5806	49	5272	94	5806	204	5272	105	5806	119	5272	167	5272	48
8	7968	102	7968	35	4019	107	7968	144	4019	120	7968	151	4019	142	4019	45
9	6587	82	6587	35	3058	87	6587	293	3058	163	6587	269	3058	131	3058	34
10	5860	101	5860	38	5552	65	5860	149	5552	146	5860	165	5552	97	5552	33

 $\Sigma si=30.0$ 

1	9668	91	9668	51	4430	104	9668	134	4430	115	9668	125	4430	142	4430	51
2	6371	120	6371	35	4119	41	6371	173	4119	89	6372	417	4119	40	4119	35
3	7149	58	7149	38	3215	120	7149	325	3215	110	7149	315	3215	106	3215	73
4	8097	271	6909	29	6840	88	6909	133	6840	111	6909	159	6840	65	6840	38
5	8930	109	8930	50	4580	112	8930	122	4580	135	8932	180	4580	91	4580	49
6	6282	107	6282	50	5307	145	6282	385	5307	121	6284	426	5307	160	5307	49
7	6137	97	6137	52	5307	111	6137	140	5307	187	6137	204	5307	126	5307	48
8	7148	72	7148	42	3215	120	7148	322	3215	110	7148	312	3215	102	3215	76
9	5940	98	5940	37	4512	105	5940	130	4512	110	5940	173	4512	101	4512	34
10	7710	98	7710	62	5485	58	7710	110	5485	84	7710	145	5485	97	5485	52

 $\Sigma si=40.0$ 

1	9460	212	7720	41	4854	87	7720	191	4854	86	7720	136	4854	132	4854	34
2	7737	110	7737	36	6420	80	7737	345	6420	124	7737	209	6420	116	6420	36
3	6559	94	6559	75	2838	67	6559	367	2838	108	6559	136	2838	159	2838	49
4	7236	141	6652	39	5491	84	6652	163	5491	117	6653	150	5491	120	5491	49
5	8919	111	8919	51	5513	89	8919	162	5513	95	8919	225	5513	132	5513	38
6	6148	104	6148	43	3138	115	6148	212	3138	184	6148	244	3138	197	3138	37
7	6669	66	6669	39	3614	67	6669	282	3614	102	6671	251	3614	158	3614	38
8	6413	76	6413	33	5103	91	6413	122	5103	103	6413	119	5103	100	5103	33
9	6429	153	6002	73	4810	73	6002	210	4810	127	6002	163	4810	164	4810	33
10	7240	152	6656	39	5491	84	6656	172	5491	120	6656	113	5491	132	5491	52

 $\Sigma si=50.0$ 

1	9035	298	8334	20	7259	89	8334	120	7259	83	8334	239	7259	104	7259	31
2	5880	87	5880	52	3860	94	5880	318	3860	101	5880	212	3860	143	3860	52
3	5992	109	5992	58	4045	89	5992	163	4045	96	5992	118	4045	134	4045	42
4	5656	107	5656	43	4839	68	5656	136	4839	91	5656	263	4839	127	4839	36
5	5755	110	5755	56	4526	98	5755	110	4526	101	5755	146	4526	105	4526	49
6	8887	170	8222	21	6813	90	8222	146	6813	81	8222	334	6813	100	6813	30
7	6426	83	6426	61	4401	89	6426	167	4401	113	6426	373	4401	138	4401	48
8	9053	97	9053	33	5354	105	9053	310	5354	146	9053	207	5354	217	5354	31
9	7116	90	7116	29	3508	88	7116	149	3508	85	7116	282	3508	92	3508	29
10	5658	112	5658	43	4839	66	5658	138	4839	91	5658	265	4839	92	4839	33

## STYLE 1, VARIATION 2(10X10X10, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	9134	698	7648	229	411	117	7655	308	411	196	7665	1054	411	321
2	10103	1111	7548	166	413	91	7553	1034	413	406	7560	587	413	374
3	6601	551	6524	165	417	190	6526	572	417	250	6528	355	417	321
4	9079	1137	5509	121	413	111	5517	526	413	211	5522	513	413	308
5	5620	412	5620	92	411	105	5620	381	411	380	5620	719	411	265
6	5621	365	5621	93	412	118	5621	473	412	347	5621	762	412	307
7	9136	623	7646	223	411	110	7646	309	411	196	7665	1054	411	321
8	7363	822	5965	121	409	82	5969	480	409	475	5974	928	409	347
9	7517	405	7062	128	411	118	7072	440	411	402	7077	1033	411	324
10	6864	384	6835	140	415	117	6837	510	415	211	6838	935	415	365
<b><math>\Sigma_{si}=1.5</math></b>														
1	8075	944	5689	112	410	130	5689	456	410	330	5690	914	410	361
2	8756	583	5906	198	411	113	5915	1033	411	336	5922	923	411	370
3	7362	529	6279	184	411	99	6289	447	411	351	6297	900	411	384
4	7461	470	6990	110	408	92	6995	445	408	295	7004	774	408	249
5	5858	342	5858	143	412	189	5858	438	412	284	5858	636	412	217
6	6845	442	5990	207	405	100	5998	440	405	361	6002	825	405	344
7	9660	913	6019	222	408	130	6025	540	408	271	6032	783	408	265
8	7360	529	6279	184	411	99	6289	447	411	351	6297	900	411	384
9	6898	360	5916	303	411	154	5922	456	411	349	5930	356	411	294
10	7247	584	6345	198	412	161	6358	462	412	330	6368	751	412	352
<b><math>\Sigma_{si}=2.0</math></b>														
1	6713	400	6713	84	412	142	6713	795	412	326	6713	350	412	259
2	7460	423	7272	113	412	90	7275	396	412	207	7279	705	412	274
3	7800	462	7523	137	410	107	7529	469	410	187	7533	739	410	313
4	8652	671	5683	182	412	74	5689	308	412	286	5692	729	412	299
5	5640	318	5621	64	404	59	5623	306	404	273	5625	665	404	305
6	5928	272	5834	99	409	103	5838	347	409	406	5841	560	409	209
7	9358	917	5980	73	410	95	5984	590	410	243	5984	592	410	237
8	9038	505	7604	293	416	77	7614	459	416	159	7619	665	416	303
9	7480	502	6920	184	407	100	6932	803	407	355	6940	610	407	255
10	7460	423	7272	113	412	90	7275	396	412	207	7279	705	412	274
<b><math>\Sigma_{si}=5.0</math></b>														
1	7138	352	6791	130	411	66	6799	762	411	235	6800	480	411	333
2	8009	415	6638	89	414	80	6644	294	414	217	6646	209	414	156
3	7804	453	6081	99	408	62	6084	242	408	234	6085	370	408	185
4	5528	296	5528	115	406	59	5528	546	406	302	5528	398	406	256
5	6506	280	6506	61	409	94	6506	241	409	140	6506	253	409	199
6	7589	314	7475	157	407	66	7486	244	407	314	7489	469	407	218
7	9409	373	8508	121	409	72	8520	298	409	138	8521	229	409	232
8	5604	238	5604	72	409	72	5604	295	409	265	5604	333	409	155
9	5530	296	5530	115	406	59	5530	546	406	302	5530	398	406	256
10	6987	389	5935	84	404	133	5944	224	404	160	5945	309	404	204

**$\Sigma si=10.0$** 

1	7040	305	6121	92	406	84	6129	284	406	243	6132	215	406	120
2	6885	325	6538	70	403	65	6547	413	403	209	6550	234	403	285
3	10120	458	5537	90	411	51	5539	226	411	179	5539	211	411	147
4	8590	392	5596	92	407	75	5597	236	407	227	5598	245	407	212
5	8786	380	6307	86	416	80	6318	247	416	216	6320	265	416	274
6	7657	383	5697	101	409	81	5701	296	409	336	5701	248	409	156
7	9898	548	6667	77	403	76	6676	289	403	146	6677	523	403	144
8	6403	331	6126	95	410	67	6133	230	410	293	6134	204	410	213
9	6534	291	6409	118	408	65	6415	255	408	194	6416	448	408	170
10	8592	399	5596	96	407	75	5597	238	407	227	5598	245	407	210

 **$\Sigma si=20.0$** 

1	7967	254	6562	44	406	67	6571	118	406	102	6571	307	406	107
2	7471	178	7305	87	408	71	7308	252	408	270	7309	462	408	156
3	6332	241	6332	99	406	76	6332	187	406	180	6332	284	406	200
4	6088	147	5843	94	411	62	5853	365	411	292	5853	476	411	152
5	5940	250	5940	97	408	60	5940	310	408	140	5940	368	408	159
6	8318	451	6279	88	406	89	6285	255	406	125	6285	420	406	181
7	5806	229	5806	53	408	57	5806	321	408	277	5806	203	408	195
8	7968	244	6562	46	406	67	6571	120	406	105	6571	307	406	107
9	6587	264	6233	78	409	65	6237	339	409	127	6237	215	409	157
10	5860	201	5860	107	409	60	5860	308	409	216	5860	298	409	145

 **$\Sigma si=30.0$** 

1	9668	373	6779	59	408	60	6788	230	408	281	6788	385	408	313
2	6372	237	6367	67	407	66	6372	303	407	219	6372	215	407	170
3	7149	251	5747	116	405	54	5754	222	405	138	5754	477	405	216
4	8097	443	6307	128	408	63	6317	241	408	264	6318	197	408	123
5	8932	259	6626	128	405	147	6635	373	405	269	6635	205	405	185
6	6284	297	5486	99	409	87	5489	296	409	235	5489	437	409	245
7	6137	249	6137	128	406	136	6137	520	406	252	6137	343	406	173
8	7148	246	5748	121	405	54	5756	236	405	142	5756	477	405	216
9	5940	250	5940	97	408	60	5940	310	408	140	5940	368	408	159
10	7710	314	6170	58	407	99	6179	235	407	243	6179	235	407	239

 **$\Sigma si=40.0$** 

1	9460	402	6664	157	405	129	6677	423	405	256	6677	228	405	314
2	7737	269	6521	78	405	134	6529	262	405	215	6529	204	405	298
3	6559	167	6491	75	409	124	6496	339	409	292	6496	232	409	271
4	7236	384	6008	50	407	125	6011	324	407	236	6011	177	407	243
5	8919	275	6560	123	410	85	6568	379	410	198	6568	217	410	281
6	6148	313	5958	89	412	125	5966	246	412	190	5966	392	412	215
7	6671	251	5951	84	408	107	5958	232	408	338	5959	245	408	365
8	6413	218	6212	100	404	97	6223	250	404	288	6223	206	404	264
9	6429	385	5486	120	411	126	5495	309	411	261	5495	251	411	169
10	7240	362	6008	56	407	125	6012	332	407	236	6011	198	407	245

 **$\Sigma si=50.0$** 

1	9035	451	5699	68	404	69	5711	261	404	180	5711	202	404	219
2	5880	78	5880	58	408	100	5880	186	408	181	5880	271	408	149
3	5992	255	5992	76	408	123	5992	487	408	198	5992	223	408	266
4	5656	187	5656	90	406	104	5656	331	406	281	5656	341	406	223
5	5755	206	5725	99	408	161	5728	217	408	226	5728	166	408	329
6	8891	309	7001	128	405	86	7012	217	405	163	7012	151	405	232
7	6426	150	6426	68	408	112	6426	310	408	165	6426	298	408	285
8	9053	256	6525	192	411	89	6537	478	411	144	6537	189	411	173
9	7116	270	5965	151	410	95	5974	406	410	251	5974	218	410	317
10	5658	192	5658	92	406	104	5658	331	406	281	5658	321	406	225

## STYLE 2, VARIATION 2(10X10X10, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	9119	270	7642	56	407	53	7649	119	407	75	7665	210	407	128
2	10096	413	7532	56	408	74	7537	129	408	111	7560	652	408	114
3	6583	158	6507	59	412	76	6509	138	412	131	6528	337	412	125
4	9053	552	5494	42	403	54	5503	125	403	79	5522	655	403	125
5	5603	109	5603	46	405	50	5603	127	405	159	5620	346	405	89
6	5603	109	5603	45	406	55	5603	129	406	162	5321	265	406	111
7	9116	270	7642	56	407	53	7649	119	407	75	7665	210	407	128
8	7349	339	5953	36	404	66	5956	123	404	86	5974	692	404	109
9	7501	133	7052	50	407	51	7062	125	407	115	7077	636	407	101
10	6847	125	6820	62	409	72	6820	119	409	99	6838	684	409	116
<b><math>\Sigma_{si}=1.5</math></b>														
1	8052	360	5669	37	406	68	5670	117	406	77	5690	299	406	125
2	8738	224	5900	31	406	50	5910	141	406	87	5922	599	406	119
3	7350	209	6277	74	409	75	6287	144	409	141	6297	234	409	118
4	7451	119	6988	55	408	68	6993	133	408	105	7004	568	408	117
5	5846	82	5846	57	406	63	5846	110	406	74	5858	702	406	107
6	6832	236	5984	74	403	58	5993	230	403	103	6002	324	403	147
7	9640	419	6017	63	407	82	6024	125	407	163	6032	265	407	177
8	7352	209	6277	74	409	75	6287	144	409	141	6297	234	409	118
9	6890	179	5911	53	408	59	5917	120	408	91	5930	215	408	125
10	7241	184	6344	76	411	71	6356	127	411	68	6368	254	411	122
<b><math>\Sigma_{si}=2.0</math></b>														
1	6694	113	6694	55	407	66	6694	128	407	115	6713	187	407	134
2	7445	177	7267	52	408	55	7271	280	408	85	7279	604	408	115
3	7785	161	7521	46	407	63	7528	139	407	50	7533	164	407	111
4	8638	426	5678	53	410	62	5684	151	410	92	5692	176	410	102
5	5625	72	5608	46	404	50	5610	237	404	50	5625	175	404	114
6	5923	120	5831	45	407	76	5835	126	407	123	5841	180	407	140
7	9348	469	5974	49	405	60	5978	174	405	62	5984	493	405	90
8	9031	246	7600	78	413	67	7611	143	413	156	7619	222	413	156
9	7476	230	6917	68	406	80	6929	129	406	91	6940	215	406	105
10	7445	177	7267	52	408	55	7271	280	408	85	7279	604	408	115
<b><math>\Sigma_{si}=5.0</math></b>														
1	7121	194	6783	62	408	66	6791	142	408	127	6800	387	408	154
2	8005	176	6636	42	412	64	6642	134	412	77	6646	344	412	101
3	7797	224	6081	40	408	50	6084	125	408	114	6085	128	408	90
4	5526	102	5526	44	406	67	5526	129	406	98	5528	227	406	140
5	6498	67	6498	51	408	67	6498	133	408	96	6506	178	408	107
6	7579	124	7473	37	407	67	7484	308	407	92	7489	378	407	113
7	9404	237	8507	60	409	50	8519	136	409	136	8521	351	409	88
8	5603	108	5603	41	409	72	5603	118	409	88	5604	136	409	104
9	5528	102	5528	44	406	67	5528	129	406	98	5528	227	406	140
10	6987	187	5935	46	404	57	5944	139	404	108	5945	142	404	94

$\Sigma si=10.0$ 

1	7040	227	6115	55	406	69	6123	118	406	98	6132	172	406	104
2	6877	195	6533	67	403	68	6542	147	403	68	6550	364	403	100
3	10113	564	5537	37	411	69	5539	134	411	63	5539	163	411	82
4	8590	315	5596	53	407	70	5597	130	407	145	5598	171	407	124
5	8779	240	6307	65	416	65	6318	115	416	98	6320	218	416	122
6	7657	415	5697	64	409	66	5701	140	409	77	5701	391	409	102
7	9898	411	6665	53	403	66	6674	125	403	98	6677	312	403	120
8	6403	126	6126	54	410	62	6133	151	410	92	6134	133	410	126
9	6531	128	6407	47	408	61	6413	120	408	76	6416	146	408	129
10	8592	315	5596	53	407	72	5597	133	407	124	5598	147	407	124

 $\Sigma si=20.0$ 

1	7967	254	6562	44	406	67	6571	118	406	102	6571	307	406	107
2	7471	145	7304	79	408	88	7308	176	408	163	7309	155	408	136
3	6332	122	6332	71	406	72	6332	154	406	102	6332	321	406	137
4	6088	86	5843	35	411	54	5853	127	411	123	5853	125	411	115
5	5940	92	5940	38	408	55	5940	160	407	75	5940	158	407	104
6	8316	371	6279	67	406	70	6285	131	406	82	6285	143	406	117
7	5806	107	5806	45	408	64	5806	134	408	92	5806	88	408	127
8	7968	245	6562	46	406	67	6571	120	406	102	6571	307	406	121
9	6587	128	6233	59	409	69	6237	149	409	110	6237	153	409	122
10	5860	106	5860	42	408	52	5860	123	408	96	5860	151	408	93

 $\Sigma si=30.0$ 

1	9668	223	6777	44	407	49	6786	129	407	108	6788	126	407	124
2	6372	119	6367	35	407	48	6371	267	407	136	6372	159	407	112
3	7149	148	5747	82	405	128	5754	141	405	193	5754	197	405	156
4	8097	429	6307	46	408	63	6317	125	408	70	6318	120	408	116
5	8930	288	6626	57	405	71	6635	127	405	127	6635	137	405	130
6	6282	203	5486	49	409	67	5489	149	409	86	5489	141	409	119
7	6137	113	6137	50	405	66	6137	117	405	112	6137	218	405	107
8	7148	152	5748	82	405	128	5756	146	405	193	5756	189	405	156
9	5940	92	5940	38	408	55	5940	160	407	75	5940	158	407	104
10	7710	206	6170	49	407	78	6179	137	407	112	6179	123	407	80

 $\Sigma si=40.0$ 

1	9460	426	6664	57	405	64	6677	139	405	92	6677	130	405	104
2	7737	153	6521	39	405	51	6529	147	405	73	6529	134	405	86
3	6559	109	6491	73	409	66	6496	135	409	73	6496	130	409	108
4	7236	210	6008	43	407	68	6011	127	407	75	6011	360	407	159
5	8919	262	6560	56	410	80	6568	126	410	78	6568	132	410	134
6	6148	147	5958	63	412	80	5966	146	412	171	5966	152	412	141
7	6669	204	5949	53	408	56	5957	126	408	100	5959	143	408	133
8	6413	145	6212	32	404	46	6223	151	404	85	6223	125	404	112
9	6429	321	5486	65	411	61	5495	119	411	111	5495	120	411	126
10	7240	212	6009	43	407	72	6011	128	407	75	6011	225	407	162

 $\Sigma si=50.0$ 

1	9035	71	5699	31	404	55	5711	143	404	69	5711	155	404	96
2	5880	100	5880	43	408	72	5880	116	408	78	5880	129	408	83
3	5992	105	5992	52	407	74	5992	146	407	131	5992	178	407	105
4	5656	89	5656	61	406	58	5656	145	406	61	5656	218	406	117
5	5755	145	5725	59	408	50	5728	115	408	98	5728	129	408	138
6	8891	388	7001	45	405	57	7012	124	405	82	7012	140	405	98
7	6426	101	6426	46	408	63	6426	146	408	76	6426	192	408	103
8	9053	268	6525	56	411	78	6537	137	411	108	6537	132	411	127
9	7116	230	5965	42	410	61	5974	158	410	88	5974	142	410	96
10	5658	102	5658	61	406	62	5658	152	406	61	5658	210	406	117



## STYLE 1, VARIATION 1(20X20X20, $\Sigma_{cap}=2.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
$\Sigma_{si}=1.0$																
1	39254	4414	24250	2150	33546	3224	24250	2672	33546	1354	24250	2681	33546	726	24250	309
2	40624	3917	23346	561	38411	2509	23346	738	38411	1487	23346	1292	38411	1841	23346	380
3	38280	1156	26296	1786	37739	2933	26296	2218	37739	1113	26296	3709	37739	1085	26296	309
4	36590	3519	24017	1825	32939	2627	24017	6659	32939	975	24017	2882	32939	1060	24017	230
5	31584	3033	18580	1647	31406	1597	18580	3093	31406	1482	18580	4836	31406	1320	18580	323
6	33479	3534	26115	1759	32726	2917	26115	2638	32726	1877	26115	3607	32726	2110	26115	353
7	32174	6671	14363	811	29001	2535	14363	6310	29001	1362	14363	2081	29001	1346	14363	253
8	30618	3278	22346	606	26246	2693	22346	1998	26246	1589	22346	671	26246	1007	22346	320
9	27710	3582	22336	2154	26943	2769	22336	2608	26943	1370	22336	2655	26943	1477	22336	269
10	27688	2820	19593	2283	24207	2293	19593	3660	24207	1040	19593	4476	24207	977	19593	377
$\Sigma_{si}=1.5$																
1	30854	2979	19986	1426	29345	2483	19986	2911	29345	1064	19986	2055	29345	756	19986	282
2	30369	3453	18275	569	26066	2800	18275	2460	26066	790	18275	1504	26066	1287	18275	427
3	32958	3239	23402	609	29671	2184	23402	1796	29671	1118	23402	1028	29671	1016	23402	336
4	36978	3111	26183	1971	33671	2440	26183	4628	33671	1099	26183	2003	33671	1222	26183	381
5	35518	1813	19962	447	35435	2051	19962	840	35435	1286	19962	686	35435	1490	19962	275
6	35435	3507	26367	1869	33574	2229	26367	3751	33574	1188	26367	3127	33574	1066	26367	263
7	34090	4032	22055	2341	32404	2580	22055	3698	32405	1079	22055	3911	32405	978	22055	231
8	27704	2910	21277	1540	27460	2010	21277	2818	27460	1022	21277	1096	27460	1059	21277	274
9	40629	3917	23346	561	38411	2509	23346	738	38411	1487	23346	1292	38411	1841	23346	380
10	28615	3093	22311	1598	26740	2180	22311	3172	26740	879	22311	1896	26740	732	22311	229
$\Sigma_{si}=2.0$																
1	31243	2168	26667	702	29297	1580	26667	1929	29297	1057	26667	2026	29297	806	26667	209
2	38283	1156	26296	1786	37739	2933	26296	2218	37739	1113	26296	3709	37739	1085	26296	309
3	32170	2152	16172	1739	30861	1538	16172	2531	30861	1099	16172	2158	30861	754	16172	289
4	34146	965	23850	2585	33524	626	23850	1725	33524	299	23850	1774	33524	326	23850	105
5	35863	3198	28557	1238	35257	2780	28557	2446	35257	1031	28557	1238	35257	1513	28557	389
6	27582	3025	20331	1985	27354	2064	20331	6672	27354	972	20331	2168	27354	920	20331	349
7	24151	2088	18779	370	22665	2758	18779	2406	22665	984	18779	1404	22665	1008	18779	265
8	36138	3538	24336	1442	33368	2084	24366	2447	33368	1383	24366	2978	33368	1268	24366	223
9	30747	2349	23491	813	30730	1391	23491	6934	30730	968	23491	749	30730	814	23491	226
10	30986	2971	22935	1560	29059	2153	22935	3322	29059	832	22935	1648	29059	610	22935	287
$\Sigma_{si}=5.0$																
1	32669	1698	19024	619	31735	1075	19024	3308	31735	560	19024	2077	31735	563	19024	145
2	36121	1384	23686	785	34157	1585	23686	2164	34157	724	23686	1566	34157	628	23686	260
3	35617	2960	28770	1510	31416	1405	28770	1736	31416	577	28770	1597	31416	601	28770	166
4	33509	1814	24129	194	30998	1812	24129	1332	30998	477	24129	2335	30998	533	24129	113
5	34714	1828	22817	1789	32516	1775	22817	2554	32516	696	22817	1346	32516	663	22817	136
6	38284	2360	26296	1786	37739	2933	26296	2218	37739	1113	26296	3709	37739	1085	26296	309
7	36967	2326	25472	924	35229	1520	25472	2053	35229	548	25472	2281	35229	665	25472	206
8	34917	1553	24013	566	32526	1603	24013	2523	32526	874	24013	915	32526	566	24013	135
9	28146	2088	16001	675	24086	1363	16001	2769	24086	726	16001	1109	24086	570	16001	185
10	38890	1945	30149	1971	36234	1541	30149	2802	36234	852	30149	1615	36234	697	30149	162

$\Sigma si=10.0$ 

1	30004	1056	21255	179	29962	872	21255	1052	29962	406	21255	1431	29962	685	21255	54
2	45447	2643	31449	415	41639	1870	31449	1512	41639	447	31449	902	41639	580	31449	160
3	32874	3232	18091	2119	28260	762	18091	2306	28260	371	18091	1530	28260	442	18091	142
4	25009	2074	17699	397	22662	1416	17699	1705	22662	502	17699	1221	22662	603	17699	151
5	30654	1357	18965	605	28391	768	18695	1978	28391	389	18965	1208	28391	571	18965	106
6	29343	2400	15808	747	25545	1593	15808	1912	25545	819	15808	1382	25545	718	15808	136
7	33510	1814	24129	194	30998	1812	24129	1332	30998	477	24129	2335	30998	533	24129	113
8	34897	1754	23030	574	31739	947	23030	1629	31739	550	23030	1661	31739	612	23030	113
9	28986	1467	17830	221	27078	820	17830	907	27078	438	17830	1731	27078	555	17830	105
10	29286	1704	20143	1973	24876	1573	20143	2314	24786	443	20143	1062	24786	605	20143	127

 $\Sigma si=20.0$ 

1	26468	1356	21240	2146	26265	1188	21240	2277	26265	505	21240	1448	26265	472	21240	128
2	24250	1258	15771	733	23017	1339	15771	1500	23017	451	15771	2286	23017	382	15771	203
3	35130	1660	18698	750	31167	1414	18698	2023	31167	460	18698	1213	31167	417	18698	117
4	31387	1015	25864	1704	28708	1245	25864	2405	28708	367	25864	1846	28708	610	25864	110
5	32971	1133	24225	1653	29117	1423	24225	3804	29117	501	24225	1541	29117	421	24225	214
6	27546	847	24234	1881	27182	1259	24234	2404	27182	337	24234	1201	27182	492	24234	270
7	25010	2074	17699	397	22662	1416	17699	1705	22662	502	17699	1221	22662	603	17699	151
8	38457	1665	30015	1983	34911	1103	30015	1386	34911	376	30015	663	34911	407	30015	118
9	34523	964	27700	420	33874	1183	27700	630	33874	400	27700	965	33874	665	27700	107
10	28884	923	20869	591	28603	850	20869	2380	28603	291	20869	1162	28603	496	20869	101

 $\Sigma si=30.0$ 

1	39890	1022	26157	1148	38810	1330	26157	1773	38810	474	26157	1565	38810	543	26157	161
2	34332	2129	25149	1374	30073	1071	25149	2089	30073	316	25149	1931	30073	395	25149	115
3	39153	1541	27504	2582	36163	1391	27504	2138	36163	312	27504	1653	36163	462	27504	143
4	30229	940	26122	843	29836	1127	26122	1795	29837	430	26122	1603	29387	382	26122	114
5	34578	1679	25179	3024	31293	1141	24980	2985	31293	400	24980	1320	31293	476	24980	177
6	40967	2856	24396	208	37882	1509	24396	1171	37882	384	24396	1191	37882	765	24396	98
7	34664	1465	25007	619	30865	802	25007	2107	30865	356	25007	1334	30865	474	25007	119
8	37531	1024	24836	1585	37267	584	24836	1854	37267	412	24836	1313	37267	329	24836	110
9	35130	1660	18698	750	31167	1414	18698	2023	31167	460	18698	1213	31167	417	18698	117
10	31973	1786	23618	1038	27537	1454	23618	1412	27537	396	23618	1894	27537	607	23618	164

 $\Sigma si=40.0$ 

1	42010	2417	34832	702	39593	1564	34832	1855	39593	550	34832	1072	39593	358	34832	108
2	26090	1332	16975	317	24886	1851	16975	795	24886	474	16975	968	24886	618	16975	101
3	31519	1106	20848	684	30815	918	20848	2760	30815	398	20848	1259	30815	459	20848	117
4	18635	1454	9225	221	16336	1074	9225	1719	16336	420	9225	1354	16336	593	9225	250
5	33810	1250	23307	1089	31863	957	23307	1554	31863	389	23307	787	31863	377	23307	111
6	33224	1375	22492	2340	30599	1114	22492	2465	30599	408	22492	1446	30599	489	22492	132
7	34331	2129	25149	1374	30073	1071	25149	2089	30073	316	25149	1931	30073	395	25149	115
8	30377	1383	17419	221	28623	974	17419	1325	28623	363	17419	1118	28623	614	17419	93
9	38737	1228	30084	325	38014	1068	30084	1175	38014	556	30084	933	38014	537	30084	139
10	35253	2052	19472	931	32499	1289	19472	2434	32499	533	19472	1355	32499	594	19472	248

 $\Sigma si=50.0$ 

1	32816	1724	26611	1541	31007	1013	26611	2459	31007	376	26611	1851	31007	382	26611	108
2	36136	1283	22405	282	33451	1456	22405	1257	33451	438	22405	1712	33451	503	22405	94
3	34881	1226	25596	448	34273	504	25596	2125	34273	408	25596	1177	34273	469	25596	93
4	31262	1170	18398	112	31020	918	18398	1068	31020	387	18398	1607	31020	675	18398	3
5	25294	1309	16678	1823	23225	980	16678	1759	23225	393	16678	1527	23225	466	16678	102
6	30099	1506	20258	1260	27980	617	20258	2387	27980	310	20258	1262	27980	561	20258	251
7	32782	1535	25432	214	30683	1003	25432	1851	30683	414	25432	1590	30683	340	25432	80
8	35721	2060	23719	751	33361	843	23719	3264	33361	290	23719	1290	33361	411	23719	122
9	18636	1454	9225	221	16336	1074	9225	1719	16336	420	9225	1354	16336	593	9225	250
10	29747	1154	20943	510	29406	1163	20943	2024	29406	384	20943	1564	29406	654	20943	103



## STYLE 2, VARIATION 1(20X20X20, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	39243	1702	24224	241	33534	297	24224	565	33534	560	24233	586	33534	495	24224	86
2	40619	893	23334	256	38408	269	23334	249	38408	447	23334	383	38408	475	23334	61
3	38282	666	26279	285	37737	304	26279	393	37737	493	26281	373	37737	656	26279	79
4	36580	646	23990	267	32929	283	23990	467	32929	534	24002	506	32929	547	23990	77
5	31577	446	18556	243	31398	297	18556	565	31398	231	18566	877	31398	575	18556	123
6	33470	658	26094	226	32720	402	26094	488	32720	732	26104	501	32720	794	26094	87
7	32169	1600	14347	141	28995	220	14347	360	28995	537	14348	599	28995	648	14347	128
8	30609	916	22321	243	26238	247	22321	255	26238	489	22329	481	26328	538	22321	57
9	27703	612	22313	313	26937	366	22313	477	26937	642	22324	642	26937	511	22313	78
10	27679	687	19569	261	24197	197	19569	401	24197	523	19577	618	24197	574	19569	79
<b><math>\Sigma_{si}=1.5</math></b>																
1	30851	727	19972	238	29342	345	19972	590	29342	526	19975	515	29342	584	19972	115
2	30361	1327	18259	175	26061	220	18259	194	26061	658	18267	263	26061	663	18259	61
3	32955	583	23392	350	29667	296	23392	200	29668	368	23392	907	29668	668	23392	211
4	36974	701	26170	190	33667	218	26170	485	33667	391	26172	462	33667	623	26170	75
5	35515	322	19944	155	35482	267	19944	746	35432	402	19953	261	35432	480	19944	48
6	35433	917	26353	314	33573	245	26353	396	33573	562	26354	595	33573	502	26353	84
7	34088	1351	22042	302	32400	393	22042	1125	32400	348	22045	1206	32400	516	22042	85
8	27696	381	21258	214	27452	305	21258	1096	27452	554	21271	596	27452	724	21258	124
9	40615	893	23334	256	38410	269	23334	249	38410	447	23334	383	38410	475	23334	61
10	28607	445	22287	279	26732	272	22287	618	26732	518	22300	676	26732	597	22287	113
<b><math>\Sigma_{si}=2.0</math></b>																
1	31240	561	26659	289	29293	284	26659	811	29293	606	26662	372	29293	529	26659	94
2	38282	666	26279	285	37737	304	26279	393	37737	493	26281	373	37737	656	26279	79
3	32168	333	16162	250	30859	267	16162	424	30859	543	16163	434	30859	619	16162	113
4	34140	449	23850	251	33524	309	23850	1245	33524	478	23850	561	33524	500	23850	132
5	35858	652	28532	256	35252	335	28532	299	35252	568	28546	466	35252	584	28532	87
6	27578	755	20318	278	27351	341	20318	404	27351	484	20323	416	27351	567	20318	85
7	24148	540	18774	245	22662	254	18774	167	22662	531	18774	311	22662	491	18774	235
8	36133	1011	24361	257	33363	287	24361	389	33363	339	24361	385	33363	548	24361	100
9	30745	486	23475	234	30727	320	23475	426	30727	370	23478	269	30727	418	23475	55
10	30985	592	22931	243	29059	284	22931	501	29059	540	22931	558	29059	538	22931	70
<b><math>\Sigma_{si}=5.0</math></b>																
1	32266	611	19016	271	31732	308	19016	471	31732	512	19020	753	31732	556	19016	116
2	36121	775	23683	180	34157	295	23683	362	34157	576	23684	928	34157	598	23683	77
3	35615	1040	28766	263	31414	311	28766	582	31414	527	28769	900	31414	465	28766	88
4	33508	424	24126	374	30996	236	24126	1194	30997	734	24128	387	30997	448	24126	56
5	34714	900	22815	200	32515	319	22815	502	32515	365	22815	453	32515	392	22815	79
6	38280	666	26279	285	37737	304	26279	393	37737	493	26281	373	37737	656	26279	79
7	36965	652	25466	192	35226	315	25466	376	35227	504	25467	524	35227	320	25466	78
8	34917	736	24012	230	32526	313	24012	315	32526	565	24012	350	32526	434	24012	82
9	28146	477	15999	253	24085	217	15999	349	24085	578	15999	493	24085	689	15999	90
10	38889	1053	30142	154	36233	248	30142	436	36233	638	30146	419	36233	500	30142	106

$\Sigma si=10.0$ 

1	30003	376	21249	246	29961	254	21249	226	29961	476	21249	294	29961	619	21249	57
2	45447	2376	31446	267	41639	292	31446	465	41639	591	31446	461	41639	1264	31446	101
3	32873	2011	18091	276	28260	299	18091	383	28260	564	18091	508	28260	510	18091	88
4	25008	601	17698	190	22661	232	17698	124	22661	550	17698	263	22661	625	17698	55
5	30654	553	18965	245	28391	308	18965	488	28391	838	18965	623	28391	499	18965	79
6	29342	1745	15804	204	25543	226	15804	535	25543	492	15806	547	25543	456	15804	78
7	33511	425	24126	374	30996	236	24126	1194	30997	734	24128	387	30997	448	24126	56
8	34897	1059	23029	276	31739	353	23029	428	31739	534	23029	645	31739	785	23029	113
9	28986	472	17829	21	27078	223	17829	208	27078	508	17829	306	27078	747	17829	184
10	29285	561	20143	319	24785	271	20143	325	24785	531	20143	380	24785	523	20143	92

 $\Sigma si=20.0$ 

1	26468	437	21240	239	26265	289	21240	1189	26265	552	21240	623	26625	855	21240	88
2	24250	623	15771	271	23017	318	15771	466	23017	382	15771	474	23017	505	15771	118
3	35132	1162	18698	198	31167	282	18698	434	31167	547	18698	532	31167	456	18698	86
4	31387	737	25864	269	28708	275	25864	376	28708	604	25864	660	28708	438	25864	107
5	32971	1169	24223	216	29117	228	24223	367	29117	580	24224	542	29117	567	24223	113
6	27546	547	24234	259	27182	284	24234	511	27182	461	24234	601	27182	650	24234	78
7	25010	601	17698	190	22661	232	17698	124	22661	550	17699	263	22661	625	17698	55
8	38456	1364	30015	307	34911	279	30015	556	34911	364	30015	427	34911	780	30015	77
9	34523	527	27700	293	33874	325	27700	179	33874	842	27700	255	33874	725	27700	57
10	28881	384	20858	205	28599	223	20858	410	28599	467	20867	1318	28599	631	20858	80

 $\Sigma si=30.0$ 

1	39890	572	26152	224	38810	306	26152	384	38810	612	26152	485	38810	558	26152	98
2	34331	921	25147	251	30073	232	25147	417	30073	475	25149	353	30073	446	25147	199
3	39152	535	27501	242	36163	251	27501	1070	36163	519	27503	543	36163	422	27501	92
4	30299	390	26122	284	29836	293	26122	1170	29837	398	26122	430	29837	595	26122	89
5	34577	1256	24975	311	31292	357	24975	486	31292	613	24977	600	31292	482	24795	82
6	40967	1636	24395	121	37882	326	24395	274	37882	442	24396	171	37882	407	24395	158
7	34664	715	25007	265	30865	229	25007	517	30865	513	25007	434	30865	455	25007	76
8	37531	478	24836	245	37267	213	24836	236	37267	271	24836	311	37267	422	24836	98
9	35130	1162	18698	198	31167	282	18698	434	31167	547	18698	532	31167	456	18698	86
10	31973	917	23614	251	27537	231	23614	933	27537	585	23614	853	27537	704	23614	90

 $\Sigma si=40.0$ 

1	42010	1719	34832	267	39593	255	34832	497	39593	662	34832	371	39593	680	34832	81
2	26090	477	16975	223	24886	217	16975	208	24886	582	16975	398	24886	526	16975	256
3	31518	419	20845	187	30814	192	20845	391	30814	632	20848	398	30814	541	20845	84
4	18635	619	9225	144	16336	204	9225	339	16336	648	9225	148	16336	533	9225	204
5	33810	1000	23307	296	31863	304	23307	599	31863	606	23307	305	31863	467	23307	86
6	33224	871	22492	246	30599	322	22492	536	30599	391	22492	466	30599	556	22492	79
7	34331	921	25147	251	30073	232	25147	417	30073	475	25149	353	30073	446	25147	199
8	30377	518	17417	170	28623	289	17417	169	28623	799	17417	321	28623	440	17417	66
9	38737	615	30084	162	38014	171	30084	145	38014	612	30084	172	38014	413	30084	68
10	35253	1280	19472	279	32499	293	19472	409	32499	398	19472	438	32499	467	19472	81

 $\Sigma si=50.0$ 

1	32816	920	26611	254	31007	306	26611	264	31007	417	26611	311	31007	580	26611	98
2	36136	595	22405	215	33450	209	22405	229	33450	575	22405	285	33450	466	22405	60
3	34881	515	25596	212	34273	261	25596	386	34273	485	25596	415	34273	480	25596	87
4	31261	525	18397	184	31020	233	18397	924	31020	798	18398	187	31020	406	18397	62
5	25294	718	16678	201	23225	279	16678	409	23225	615	16678	513	23225	596	16678	79
6	30099	565	20258	331	27979	233	20258	335	27979	678	20258	432	27979	522	20258	123
7	32782	833	25432	243	30683	260	25432	602	30683	565	25432	1066	30683	413	25432	60
8	35721	1328	23719	262	33361	295	23719	682	33361	584	23719	538	33361	704	23719	134
9	18637	619	9225	144	16336	204	9225	339	16336	648	9225	148	16336	533	9225	66
10	29747	577	20943	259	29406	225	20943	514	29406	615	20943	558	29406	585	20943	92

## STYLE 1, VARIATION 2(20X20X20, $\Sigma_{cap}=2.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	39254	27637	7965	1613	411	1815	7984	5189	411	983	7999	8269	411	1407
2	40624	141003	6774	2874	405	2055	6791	8251	405	1145	6808	9239	405	1257
3	38280	30028	7118	2616	406	1782	7137	6367	406	1039	7147	6966	406	1181
4	36590	41022	7035	2139	409	2458	7055	7261	409	1335	7072	9527	409	1544
5	31584	37775	6086	2172	407	2218	6101	4366	407	1224	6115	7466	407	2168
6	33479	28449	6593	2219	410	2217	6607	5233	410	1136	6622	9360	410	1382
7	32174	31092	6704	1950	408	1771	6719	6422	408	1610	6733	10489	408	2093
8	30618	25153	6429	2038	407	1552	6445	5221	407	1588	6461	7944	407	1533
9	27710	16325	6270	2798	406	2309	6286	3890	406	2275	6302	9038	406	1760
10	27688	13619	6510	3135	411	1701	6523	8231	411	1873	6540	8222	411	1183
<b><math>\Sigma_{si}=1.5</math></b>														
1	30854	10698	7136	1841	405	1593	7154	5723	405	1049	7165	6301	405	1234
2	30369	9991	7880	4056	407	1730	7897	3751	407	1086	7909	7092	407	1916
3	32958	22622	6633	1148	405	1850	6649	3650	405	1333	6659	5810	405	2091
4	36978	50516	7486	2061	407	2130	7504	4912	407	1244	7515	6723	407	1524
5	35518	37812	6767	1603	404	1499	6784	4688	404	2099	6793	5966	404	2060
6	35435	35331	7019	2679	404	1488	7035	7040	404	1056	7050	6911	404	1352
7	34090	24744	6680	1742	403	1524	6698	5301	403	1705	6710	7459	403	1146
8	27704	10866	6774	2963	407	1890	6789	4051	407	949	6801	7110	407	1261
9	40629	141013	6774	2874	405	2055	6791	8251	405	1145	6808	9239	405	1257
10	28615	10700	6442	1856	406	1311	6458	6055	406	1179	6471	5919	406	1283
<b><math>\Sigma_{si}=2.0</math></b>														
1	31243	10664	7138	1093	406	948	7152	5705	406	1156	7160	4940	406	1616
2	38283	30028	7118	2616	406	1782	7137	6367	406	1039	7147	6966	406	1181
3	32170	14634	6652	2351	406	1746	6667	5507	406	872	6676	4395	406	1199
4	34146	17524	6578	821	405	502	6595	1581	405	543	6595	3248	405	857
5	35863	26498	6891	1554	405	1193	6908	5514	405	975	6917	4262	405	1034
6	27582	14897	5846	784	407	1619	5861	3744	407	1734	5868	5293	407	1726
7	24151	7960	5995	1035	406	1135	6014	4022	406	1121	6021	4863	406	1340
8	36138	36557	6817	1603	405	1970	6836	3237	405	1113	6843	5657	405	1273
9	30747	10809	6995	2149	405	1632	7013	3890	405	1871	7023	5491	405	1587
10	30986	8856	7668	2134	407	2208	7684	4843	407	1319	7693	5019	407	1138
<b><math>\Sigma_{si}=5.0</math></b>														
1	32269	12025	6874	777	406	957	6887	3304	406	639	6889	4118	406	677
2	36121	15297	7257	1326	403	1060	7274	3378	403	959	7279	4654	403	676
3	35617	14010	7156	1698	405	1426	7180	4971	405	1034	7172	2965	405	1306
4	33509	12564	6510	1072	404	1696	6528	2304	404	1003	6531	3952	404	876
5	34714	13654	6789	1667	405	933	6804	3387	405	766	6807	3446	405	710
6	38284	30028	7118	2616	406	1782	7137	6367	406	1039	7147	6966	406	1181
7	36967	16807	7032	2256	405	1454	7047	2514	405	554	7051	3431	405	768
8	34917	14180	7037	1484	404	1193	7050	3152	404	718	7052	3136	404	999
9	28146	5291	6961	874	407	951	6975	2622	407	1044	6977	2895	407	975
10	38890	42899	6437	473	404	1082	6455	1808	404	616	6457	3717	404	974

$\Sigma si=10.0$ 

1	30004	7729	6872	860	403	1067	6889	1642	403	696	6892	2632	403	1148
2	45447	41199	7312	588	403	585	7330	1704	403	497	7332	2002	403	833
3	32874	12316	6511	1361	408	1382	6525	1923	408	575	6526	3104	408	778
4	25009	5603	6206	625	406	849	6221	2133	406	463	6222	2274	406	974
5	30654	6347	7061	724	405	593	7077	1988	405	766	7078	3359	405	1041
6	29343	7971	6175	995	403	1191	6190	1911	403	741	6192	3126	403	686
7	33510	12564	6510	1072	404	1696	6528	2304	404	1003	6531	3952	404	876
8	34897	14563	6260	659	405	1052	6276	3055	405	626	6277	1772	405	639
9	28986	5902	6735	965	404	1606	6753	2202	404	564	6754	2844	404	856
10	29286	10250	6050	1281	403	1069	6066	2199	403	491	6067	1741	403	654

 $\Sigma si=20.0$ 

1	26468	3899	6759	825	405	1695	6777	1705	405	490	6777	2542	405	1004
2	24250	3708	6083	543	404	852	6097	1923	404	550	6097	2450	404	762
3	35130	16114	6611	596	404	1621	6626	2082	404	594	6627	1573	404	888
4	31387	10874	6252	1661	404	1715	6270	1420	404	555	6270	2872	404	947
5	32971	7453	6695	699	403	1801	6709	2503	403	730	6709	2827	403	1308
6	27546	3513	6889	573	405	993	6905	1244	405	630	6905	1675	405	694
7	25010	5603	6206	625	406	849	6221	2133	406	463	6222	2274	406	974
8	38457	14628	7095	514	405	452	7097	1429	405	685	7111	3134	405	579
9	34523	10572	6854	791	405	613	6872	1784	405	447	6872	2835	405	729
10	28884	8571	6605	836	405	1118	6622	2067	405	939	6622	1288	405	867

 $\Sigma si=30.0$ 

1	39890	15224	7439	776	405	700	7456	1858	405	600	7458	1392	405	598
2	34332	8652	7104	696	406	410	7117	1601	406	501	7118	3174	406	679
3	39153	9195	6977	873	407	1675	6994	1960	407	562	6994	1516	407	661
4	30229	4766	7079	1101	404	872	7096	1795	404	523	7199	1311	404	583
5	34578	11206	7040	983	405	1759	7057	2722	405	574	7057	2656	405	803
6	40967	19134	6985	723	407	1547	6999	1673	407	660	7000	2238	407	528
7	34664	11544	6721	993	403	451	6737	1221	403	733	6738	2525	403	771
8	37531	13569	6597	426	403	1059	6613	2082	403	663	6613	2618	403	592
9	35130	16114	6611	596	404	1621	6626	2082	404	594	6627	1573	404	888
10	31973	4526	7440	1084	404	690	7456	2042	404	536	7458	2739	404	741

 $\Sigma si=40.0$ 

1	42010	6787	6481	909	403	420	6500	1328	403	772	6501	2486	403	807
2	26090	4050	6263	624	405	933	6277	1610	405	775	6278	2511	405	1097
3	31519	6841	7345	982	405	978	7360	2193	405	631	7360	1632	405	794
4	18635	2386	5578	215	406	1414	5588	1438	405	732	5589	1561	406	789
5	33810	6598	6549	581	405	730	6564	2350	405	496	6565	1154	405	826
6	33224	7895	6553	623	404	567	6570	2013	404	435	6570	2910	404	598
7	34331	8652	7104	696	406	410	7117	1601	406	501	7118	3174	406	679
8	30377	5689	7107	470	403	1170	7122	1511	403	501	7123	3189	403	767
9	38737	18607	6944	687	404	569	6961	2036	404	660	6962	2208	404	625
10	35253	14569	5877	492	403	578	5893	1428	403	601	5894	2581	403	761

 $\Sigma si=50.0$ 

1	32816	11232	7773	794	404	835	7789	4127	404	500	7789	1359	404	853
2	36136	16523	6747	932	406	1353	6764	1539	405	503	6764	2676	406	556
3	34881	12548	7324	766	405	1103	7343	2154	405	504	7343	2013	405	559
4	31262	7824	6006	715	405	531	6020	1461	405	533	6020	1865	405	632
5	25294	4081	5929	812	406	1009	5944	1947	406	431	5944	2540	406	661
6	30099	5278	7097	542	404	1118	7109	1472	404	514	7109	1213	404	1056
7	32782	12564	6573	457	404	886	6589	1252	404	540	6589	2550	404	731
8	35721	21598	6805	1107	405	1154	6824	2194	405	676	6824	1257	405	507
9	18636	2386	5578	215	406	1414	5588	1438	405	732	5589	1561	406	789
10	29747	68407	6152	906	405	682	6167	2117	405	463	6168	1239	405	916

## STYLE 2, VARIATION 2(20X20X20, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	39243	12924	7955	344	404	197	7974	577	404	407	7999	1095	404	403
2	40619	39111	6752	403	385	168	6783	646	385	518	6801	1396	385	435
3	38281	21886	7116	364	405	194	7135	665	405	392	7147	2456	405	590
4	36580	15204	7028	418	405	238	7047	762	405	542	7072	1842	405	481
5	31577	18622	6078	288	404	182	6093	543	404	574	6115	1720	404	350
6	33470	14398	6584	306	404	172	6598	661	404	416	6622	1290	404	333
7	32169	12132	6700	185	406	173	6715	668	406	435	6733	1434	406	481
8	30609	12397	6423	315	405	181	6439	651	405	582	6461	1483	405	438
9	27703	6625	6265	476	404	181	6281	792	404	488	6302	1537	404	692
10	27679	4835	6502	449	407	212	6515	668	407	427	6540	1290	407	478
<b><math>\Sigma_{si}=1.5</math></b>														
1	30851	6507	7133	453	404	177	7151	675	404	527	7165	2671	404	538
2	30361	5330	7876	412	405	162	7894	525	405	603	7909	2528	405	350
3	32955	12356	6630	277	405	177	6647	565	404	302	6659	2599	405	454
4	36974	19899	7482	353	406	215	7501	606	406	630	7515	1367	406	615
5	35515	20649	6764	377	403	186	6781	469	403	424	6793	1346	403	587
6	35433	17888	7017	312	403	158	7034	597	403	508	7050	2794	403	312
7	34088	15040	6678	387	402	144	6696	600	402	367	6710	2328	402	394
8	27696	5431	6773	544	405	180	6788	597	405	445	6801	928	405	395
9	40615	19112	6752	403	385	168	6783	646	385	518	6801	1396	385	435
10	28607	5843	6436	411	404	231	6453	701	404	506	6471	2966	404	645
<b><math>\Sigma_{si}=2.0</math></b>														
1	31240	8255	7136	324	406	235	7150	482	405	551	7160	1289	406	426
2	38282	21886	7116	364	405	194	7135	665	405	392	7147	2456	405	590
3	32168	8516	6651	448	405	204	6666	774	405	407	6676	2614	405	421
4	34140	12563	6578	363	405	180	6595	558	405	304	6595	823	405	373
5	35858	21106	6887	362	403	163	6904	692	403	397	6917	2686	403	497
6	27578	9230	5842	316	406	176	5858	713	406	348	5868	1221	406	468
7	24148	4813	5993	354	404	195	6012	650	404	544	6021	2759	404	373
8	36133	27361	6815	239	403	159	6833	532	403	405	6843	2267	403	375
9	30745	7119	6992	398	405	196	7011	601	405	237	7023	1125	405	379
10	30985	5232	7667	226	407	197	7684	652	407	317	7693	2330	407	427
<b><math>\Sigma_{si}=5.0</math></b>														
1	32266	10473	6871	338	405	207	6884	829	405	380	6889	987	405	506
2	36121	13503	7257	383	403	180	7274	613	403	492	7279	766	403	393
3	35615	13828	7155	440	404	167	7168	670	404	342	7172	775	404	413
4	33508	16526	6509	298	403	220	6526	542	403	234	6531	817	403	503
5	34714	18388	6789	268	405	195	6804	629	405	447	6807	2020	405	318
6	38280	20882	7116	364	405	194	7135	665	405	392	7147	2456	405	590
7	36965	15553	7029	468	405	201	7044	629	405	464	7051	632	405	449
8	34917	14750	7036	276	404	222	7049	458	404	508	7052	2149	404	453
9	28146	4594	6961	384	407	162	6975	571	407	441	6977	1946	407	443
10	38889	41599	6436	313	404	183	6454	498	404	363	6457	617	404	414

**$\Sigma si=10.0$** 

1	30003	7358	6872	325	403	187	6889	683	403	383	6892	785	403	532
2	45447	27665	7312	304	403	181	7330	601	403	367	7332	848	403	495
3	32873	12543	6511	375	407	219	6525	479	407	542	6526	686	407	503
4	25008	4911	6206	299	406	170	6220	558	406	473	6222	809	406	502
5	30654	7547	7061	261	405	159	7077	504	405	368	7078	749	405	315
6	29342	13101	6174	323	403	194	6189	483	403	517	6192	916	403	414
7	33511	16526	6509	298	403	220	6526	542	403	234	6531	817	403	503
8	34897	9675	6260	245	404	225	6276	548	404	321	6277	744	404	495
9	28986	6545	6735	248	404	144	6753	646	404	373	6754	1431	404	338
10	29285	10069	6049	211	402	148	6065	564	402	301	6067	1565	402	351

 **$\Sigma si=20.0$** 

1	26468	4286	6759	329	405	202	6777	514	405	517	6777	653	405	582
2	24250	4150	6083	205	404	169	6097	606	404	379	6097	1329	404	389
3	35132	18261	6611	324	404	214	6626	527	404	499	6627	639	404	392
4	31387	13339	6252	298	404	163	6270	601	404	599	6270	1355	404	404
5	32971	14408	6695	236	403	199	6709	687	403	807	6709	820	403	424
6	27546	4520	6889	315	405	190	6905	658	405	401	6905	640	405	449
7	25010	4911	6206	299	406	170	6220	558	406	473	6222	809	406	502
8	38456	20180	7095	297	405	170	7111	688	405	323	7111	606	405	415
9	34523	13950	6854	437	405	218	6872	459	405	748	6872	845	405	479
10	28881	9777	6605	330	404	188	6622	957	404	440	6622	753	404	476

 **$\Sigma si=30.0$** 

1	39890	20916	7439	400	405	235	7456	601	405	406	7458	917	405	410
2	34331	11369	7104	339	406	165	7117	656	406	523	7118	722	406	352
3	39152	14565	6976	312	406	170	6993	700	406	562	6994	884	406	425
4	30229	7078	7079	347	404	173	7096	647	404	407	7097	743	404	488
5	34577	15515	7040	467	405	215	7057	753	405	676	7057	603	405	513
6	40967	26950	6984	407	407	209	6999	505	407	543	7000	710	407	440
7	34664	12411	6721	404	403	203	6737	609	403	631	6738	778	403	829
8	37531	24330	6597	332	403	156	6613	530	403	319	6613	730	403	401
9	35130	18261	6611	324	404	214	6626	527	404	499	6627	639	404	392
10	31973	6601	7440	431	404	193	7456	723	404	411	7458	884	404	419

 **$\Sigma si=40.0$** 

1	42010	15647	6481	249	403	164	6500	678	403	345	6501	705	403	334
2	26090	4826	6263	334	405	212	6277	457	405	325	6278	674	405	435
3	31518	12566	7344	383	405	166	7359	667	405	457	7360	713	405	384
4	18635	2810	5578	247	406	183	5588	559	405	724	5589	820	405	829
5	33810	17031	6549	301	405	217	6564	752	405	454	6565	726	405	376
6	33224	7476	6553	315	404	183	6570	572	404	438	6570	635	404	333
7	34331	11369	7104	339	406	165	7117	656	406	523	7118	722	406	352
8	30377	6440	7107	279	403	144	7122	600	403	502	7123	597	403	347
9	38737	20787	6944	402	404	151	6961	628	404	499	6962	679	404	412
10	35253	13565	5877	213	403	222	5893	628	403	487	5894	896	403	407

 **$\Sigma si=50.0$** 

1	32816	8462	7773	363	404	254	7789	633	404	574	7789	758	404	604
2	36136	12496	6747	240	406	154	6764	484	405	512	6764	556	406	477
3	34881	18500	7324	326	405	192	7343	623	405	435	7343	692	405	374
4	31261	10250	6006	220	405	166	6019	639	405	354	6020	877	405	399
5	25294	4896	5929	275	406	157	5944	618	406	438	5944	785	406	274
6	30099	6463	7096	301	404	217	7108	679	404	604	7109	820	404	565
7	32782	7432	6573	292	404	196	6589	499	404	471	6589	656	404	557
8	35721	27972	6805	365	405	229	6824	595	405	406	6824	789	405	436
9	18635	2810	5578	247	406	183	5588	559	405	724	5589	820	405	829
10	29747	8456	6152	285	405	168	6167	469	405	418	6168	754	405	493



## STYLE 1, VARIATION 1(20X20X20, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	9528	1773	9528	314	9168	1560	9528	2344	9168	943	9528	7850	9168	904	9168	339
2	11298	3485	7475	2042	7689	1884	7475	3696	7689	642	7475	4030	7689	504	7475	245
3	12594	3416	8190	1573	8280	1068	8190	2402	8280	742	8190	2108	8280	541	8190	256
4	11202	3406	7258	1789	7479	1610	7258	4094	7479	654	7258	5735	7479	392	7258	168
5	8120	2487	6528	956	6551	1034	6528	2433	6551	551	6528	4758	6551	543	6528	357
6	11763	3965	7485	1861	7700	1694	7485	3660	7700	1251	7485	4033	7700	1190	7485	175
7	13740	4918	9196	1574	9215	1421	9196	3265	9215	919	9196	2752	9215	782	9196	302
8	11865	5198	9739	2104	9926	1716	9739	4198	9926	782	9739	3275	9926	598	9739	443
9	10340	2354	10058	2129	10136	1499	10058	3760	10136	854	10058	11892	10136	567	10058	434
10	6495	1365	6495	1214	5722	1174	6495	2890	5722	445	6495	4732	5722	355	5722	174
<b><math>\Sigma_{si}=1.5</math></b>																
1	8722	2179	7026	1449	7341	1333	7026	4037	7341	723	7026	2799	7341	570	7026	312
2	13273	2751	9740	2345	9888	1812	9740	3450	9888	585	9740	2503	9888	435	9740	258
3	12277	3185	8379	1617	8918	1362	8379	3180	8918	631	8379	3926	8918	566	8379	246
4	8861	1671	8861	506	7676	890	8861	2434	7676	974	8861	7009	7676	689	7676	312
5	12091	3023	8384	652	8500	1264	8384	1762	8500	896	8384	1785	8500	1161	8384	261
6	12098	3538	7433	1579	7433	1474	7433	2773	7433	467	7433	2889	7433	357	7433	291
7	11389	3088	6023	548	6161	479	6023	1610	6161	635	6023	1139	6161	756	6023	297
8	7596	1620	7596	460	7578	585	7596	1892	7578	793	7596	4762	7578	816	7578	236
9	11298	3485	7475	2042	7689	1884	7475	3696	7689	642	7475	4030	7689	504	7475	245
10	12082	4197	8043	1114	8261	1392	8043	2903	8261	765	8043	8696	8261	564	8043	379
<b><math>\Sigma_{si}=2.0</math></b>																
1	7317	1429	7317	408	7045	1216	7317	2035	7045	775	7317	4021	7045	743	7045	248
2	12594	3416	8190	1573	8280	1068	8190	2402	8280	742	8190	2108	8280	541	8190	256
3	11878	2796	6779	1525	7275	1398	6779	3276	7275	625	6779	2100	7275	392	6779	164
4	8898	822	6434	460	6639	1004	6434	1751	6639	384	6434	1521	6639	349	6434	141
5	11727	3081	6688	1668	7251	2063	6688	1843	7251	745	6688	2030	7251	424	6688	125
6	10884	2991	5773	1494	5774	1767	5773	2633	5774	439	5773	2994	5774	511	5773	187
7	11668	2903	10247	1789	10266	2243	10247	4216	10266	968	10247	2667	10266	1027	10247	423
8	5524	1596	5524	137	5511	1603	5524	1249	5511	1038	5524	1734	5511	976	5511	213
9	7413	1323	7128	759	7095	1021	7128	2616	7095	608	7128	2301	7095	754	7095	432
10	11267	3223	5672	386	5717	1015	5672	1190	5717	774	5672	934	5717	973	5672	357
<b><math>\Sigma_{si}=5.0</math></b>																
1	9804	1453	8695	846	8864	1230	8695	2917	8864	312	8695	2008	8864	348	8695	105
2	13565	2349	9166	554	9359	854	9166	2940	9359	606	9166	780	9359	403	9166	115
3	12952	2066	8821	1999	9330	1010	8821	2589	9330	483	8821	1305	9330	460	8821	203
4	6293	1496	6293	170	5469	1015	6293	1801	5469	375	6293	1980	5469	480	5469	113
5	11993	2506	7766	1795	7915	856	7766	1979	7915	328	7766	1120	7915	559	7766	168
6	12594	3416	8190	1573	8280	1068	8190	2402	8280	742	8190	2108	8280	541	8190	256
7	12377	2413	9030	145	9086	1215	9030	1126	9086	528	9030	671	9086	529	9030	267
8	6358	1595	6358	1812	5598	639	6358	2268	5598	390	6358	1448	5598	477	5598	219
9	11668	2903	10247	1789	10266	2243	10247	4216	10266	968	10247	2667	10266	1027	10247	423
10	10938	1981	6756	485	6903	940	6756	3514	6903	559	6756	1280	6903	601	6756	158

**Σsi=10.0**

1	6084	1224	6084	74	5859	990	6084	1713	5859	372	6084	2045	5859	464	5859	64
2	13043	1150	9909	1105	10281	1541	9909	2715	10281	250	9909	839	10281	229	9909	107
3	10796	1338	7261	1674	7366	1680	7261	2403	7366	201	7261	2403	7366	327	7261	223
4	6959	1084	6471	445	6040	959	6471	2500	6040	491	6471	1780	6040	337	6040	120
5	11249	1496	6475	1866	6610	612	6475	1860	6610	183	6475	1224	6610	249	6475	303
6	10836	2574	6168	705	6168	779	6168	2013	6168	449	6168	1838	6168	339	6168	201
7	6295	1496	6293	170	5469	1015	6293	1801	5469	375	6293	1980	5469	480	5469	113
8	6211	997	6211	108	6196	2293	6211	2095	6196	394	6211	1474	6196	388	6196	82
9	6449	1193	6449	147	5793	1340	6449	1587	5793	201	6449	1906	5793	260	5793	143
10	10782	1711	6302	358	6302	798	6302	2776	6302	338	6302	1032	6302	311	6302	119

**Σsi=20.0**

1	7034	1067	7034	78	6772	540	7034	1442	6772	432	7034	2107	6772	353	6772	40
2	11323	973	7027	197	7278	992	7027	1090	7278	415	7027	539	7278	429	7027	82
3	11912	1489	7585	2176	7688	2678	7585	2288	7688	371	7585	994	7688	248	7585	112
4	6007	891	6007	80	5221	1062	6007	1637	5221	353	6007	2011	5221	308	5221	40
5	12012	1708	7146	806	7253	1264	7146	3406	7253	247	7146	978	7253	289	7146	167
6	12135	1933	7013	541	7014	615	7013	3030	7014	272	7013	878	7014	218	7013	221
7	6960	1084	6471	445	6040	959	6471	2500	6040	491	6471	1780	6040	337	6040	120
8	8726	928	8202	310	8372	1522	8202	1792	8372	216	8202	1533	8372	354	8202	88
9	6564	799	6564	465	5778	930	6564	1494	5778	257	6564	1835	5778	238	5778	120
10	11832	1870	6938	596	6978	714	6938	1754	6978	373	6938	768	6978	269	6938	95

**Σsi=30.0**

1	9431	1243	9431	193	9074	723	9431	1942	9074	324	9431	1638	9074	2227	9074	92
2	13354	1188	8778	2346	8993	643	8778	1957	8993	264	8778	970	8993	417	8778	101
3	11577	1207	7846	1866	8245	1291	7846	2164	8245	304	7846	943	8245	509	7846	126
4	8145	758	8145	195	7059	935	8145	2411	7059	346	8145	1795	7059	423	7059	112
5	13117	1131	8334	911	8464	967	8334	2377	8464	291	8334	815	8464	258	8334	105
6	11601	1631	6416	308	6418	1184	6416	2828	6418	211	6416	1889	6418	256	6416	108
7	13571	1490	10531	270	10531	723	10537	1455	10531	269	10537	992	10531	399	10531	95
8	6028	795	6028	104	6014	847	6028	1355	6014	370	6028	666	6014	291	6014	78
9	11910	1489	7585	2176	7688	2678	7585	2288	7688	371	7585	994	7688	248	7585	112
10	13033	1853	9245	414	9286	793	9245	2221	9286	349	9245	1648	9286	337	9245	162

**Σsi=40.0**

1	7178	1011	7178	233	6910	1083	7178	1983	6910	384	7178	1982	6910	315	6910	102
2	11324	1136	7320	404	7705	715	7320	1353	7705	429	7320	593	7705	451	7320	92
3	12913	1530	9211	2056	9447	778	9211	2206	9447	397	9211	1754	9447	478	9211	269
4	5627	969	5627	115	4897	439	5627	1723	4897	246	5627	1084	4897	321	4897	78
5	13314	1535	10032	1537	10038	1621	10032	2077	10038	244	10032	1291	10038	245	10032	232
6	11579	1326	7751	209	7751	1851	7751	1774	7751	248	7751	1826	7751	206	7751	98
7	13355	1188	8778	2346	8993	643	8778	1957	8993	264	8778	970	8993	417	8778	101
8	7048	987	7048	106	7031	1335	7048	1782	7031	401	7048	973	7031	298	7031	80
9	7756	920	6395	507	6452	886	6395	1923	6452	391	6395	2180	6452	337	6395	127
10	11096	1502	6241	246	6242	481	6241	2121	6242	207	6241	912	6242	281	6241	99

**Σsi=50.0**

1	9408	1123	9408	327	9052	852	9408	1622	9052	379	9408	1753	9052	379	9052	96
2	11036	973	6856	208	7762	921	6856	1418	7762	243	6856	861	7762	317	6856	83
3	12735	1184	9131	2273	10291	1650	9131	1563	10291	304	9131	1667	10291	405	9131	136
4	5821	875	5821	46	5062	863	5821	2004	5062	319	5821	3113	5062	433	5062	3
5	10998	1051	6346	988	6350	1044	6346	1917	6350	240	6346	1160	6350	294	6346	120
6	13204	1603	7788	352	7789	1038	7788	2490	7789	272	7788	1153	7789	250	7788	136
7	11201	1500	6909	101	7070	818	6909	117	7070	349	6909	621	7070	405	6909	98
8	9931	875	9143	1452	9407	498	9143	2586	9407	270	9143	1170	9407	280	9143	226
9	5625	969	5627	115	4897	439	5627	1723	4897	246	5627	1084	4897	321	4897	78
10	10651	1905	5824	343	5947	746	5824	2013	5947	237	5824	709	5947	344	5824	146



## STYLE 2, VARIATION 1(20X20X20, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		FORMULATION 7B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>																
1	9504	360	9504	73	9144	113	9504	993	9144	438	9528	1349	9144	307	9144	74
2	11287	709	7458	74	7679	170	7458	1469	7679	544	7470	1338	7679	344	7458	71
3	12590	968	8185	216	8276	222	8185	360	8276	403	8190	505	8276	625	8185	83
4	11187	1165	7238	256	7461	271	7238	538	7461	465	7255	1271	7461	347	7238	136
5	8096	343	6506	231	6531	248	6506	815	6531	578	6527	785	6531	333	6506	79
6	11748	704	7461	217	7680	230	7461	822	7680	1054	7483	987	7680	465	7461	96
7	13725	1520	9175	195	9197	211	9175	600	9197	633	9193	725	9197	477	9175	78
8	11849	1381	9715	225	9906	233	9715	510	9906	400	9729	904	9906	673	9715	78
9	10316	486	10036	79	10121	180	10036	529	10121	377	10049	875	10121	426	10036	77
10	6461	310	6461	82	5689	193	6461	1017	5689	516	6495	805	5689	383	5689	82
<b><math>\Sigma_{si}=1.5</math></b>																
1	8704	439	7013	99	7332	261	7013	618	7332	304	7023	816	7332	280	7013	98
2	13262	790	9725	85	9876	210	9725	553	9876	409	9736	927	9876	415	9725	78
3	12270	936	8367	232	8910	267	8367	725	8910	292	8377	1042	8910	346	8367	75
4	8851	388	8851	272	7666	106	8851	1421	7666	566	8861	1674	7666	315	7666	219
5	12081	920	8366	130	8488	217	8366	271	8488	294	8381	429	8488	340	8366	55
6	12090	1219	7420	235	7420	252	7420	917	7420	235	7433	1141	7420	494	7420	89
7	11378	871	6010	102	6149	172	6010	727	6149	491	6022	399	6149	406	6010	58
8	7582	426	7582	59	7564	301	7582	1794	7564	350	7596	2558	7564	344	7564	224
9	11285	709	7458	74	7679	170	7458	1469	7679	544	7470	1338	7679	344	7458	71
10	12063	1610	8018	247	8241	176	8018	637	8241	346	8041	828	8241	382	8018	83
<b><math>\Sigma_{si}=2.0</math></b>																
1	7307	363	7307	147	7035	169	7307	481	7035	549	7317	1728	7035	374	7035	237
2	12590	968	8185	216	8276	222	8185	360	8276	403	8190	505	8276	625	8185	83
3	11872	681	6769	239	7267	272	6769	592	7267	493	6778	912	7267	414	6769	107
4	8898	591	6434	191	6638	289	6434	752	6638	479	6434	670	6638	356	6434	103
5	11724	764	6682	317	7247	202	6682	554	7247	300	6687	634	7247	450	6682	81
6	10879	1084	5765	206	5765	233	5765	880	5765	237	5773	672	5765	389	5765	87
7	11665	1125	10239	209	10262	338	10239	649	10262	435	10243	660	10262	373	10239	84
8	5519	329	5519	262	5509	271	5519	309	5505	176	5524	450	5505	630	5505	254
9	7400	364	7123	98	7085	270	7123	784	7085	457	7126	747	7085	373	7085	58
10	11252	1463	5656	194	5702	132	5656	664	5702	342	5671	893	5702	438	5656	63
<b><math>\Sigma_{si}=5.0</math></b>																
1	9793	349	8687	107	8858	176	8687	525	8858	319	8695	1298	8858	230	8687	101
2	13563	1257	9159	62	9354	135	9159	398	9354	213	9164	493	9354	336	9159	91
3	12951	1104	8819	271	9328	298	8819	608	9328	486	8821	834	9328	347	8819	116
4	6291	383	6291	113	5466	111	6291	1610	5466	575	6293	1422	5466	374	5466	64
5	11985	822	7755	234	7906	172	7755	449	7906	343	7765	429	7906	376	7755	96
6	12590	968	8185	216	8276	222	8185	360	8276	403	8190	505	8276	625	8185	83
7	12373	1383	9030	139	9086	300	9030	426	9086	417	9030	301	9086	754	9030	58
8	7017	394	7017	63	7000	192	7017	495	7000	257	7020	498	7000	410	7000	100
9	11658	1125	10239	209	10262	338	10239	649	10262	435	10243	660	10262	373	10239	84
10	10936	1660	6752	139	6900	265	6752	408	6900	249	6756	1239	6900	475	6752	96

$\Sigma si=10.0$ 

1	6078	387	6078	63	5854	159	6078	1771	5854	306	6084	560	5854	427	5854	65
2	13042	1015	9906	116	10280	186	9906	882	10280	263	9908	511	10280	316	9906	114
3	10796	556	7261	240	7366	215	7261	619	7366	570	7261	1077	7366	405	7261	112
4	6958	437	6461	200	6039	138	6461	559	6039	516	6471	696	6039	366	6039	59
5	11249	712	6475	196	6610	238	6475	477	6610	523	6475	405	6610	366	6475	103
6	10830	1252	6164	200	6164	274	6164	769	6164	450	6168	715	6164	373	6164	79
7	6290	385	6291	113	5466	111	6291	1610	5466	575	6293	1422	5466	374	5466	64
8	6211	363	6211	52	6196	121	6211	559	6196	350	6211	1553	6196	396	6196	51
9	6448	261	6448	70	5793	227	6448	725	5793	398	6449	579	5793	351	5793	76
10	10781	1361	6302	157	6302	275	6307	510	6302	262	6307	719	6302	493	6302	89

 $\Sigma si=20.0$ 

1	7034	336	7034	59	6772	183	7034	412	6772	477	7034	587	6772	303	6772	56
2	11323	1001	7027	60	7278	157	7027	342	7278	427	7027	256	7278	427	7027	55
3	11912	659	7585	247	7688	350	7585	659	7688	322	7585	936	7688	427	7585	72
4	6007	217	6007	78	5221	123	6007	624	5221	596	6007	593	5221	351	5221	47
5	12011	893	7144	206	7251	275	7144	477	7251	212	7146	669	7251	483	7144	122
6	12135	1169	7012	222	7014	250	7012	523	7014	338	7013	414	7014	431	7012	84
7	6956	446	6461	200	6039	138	6461	559	6039	516	6471	696	6039	366	6039	59
8	8726	477	8202	90	8372	181	8202	650	8372	291	8202	444	8372	286	8202	84
9	6564	348	6564	80	5778	257	6564	282	5778	226	6564	958	5778	332	5778	82
10	11832	1201	6938	196	6978	203	6938	609	6978	459	6938	1007	6978	343	6938	103

 $\Sigma si=30.0$ 

1	9431	576	9431	133	9074	151	9431	1541	9074	324	9431	1379	9074	268	9074	156
2	13353	784	8776	105	8992	212	8776	769	8992	415	8778	707	8992	400	8776	94
3	11577	745	7846	272	8245	316	7846	807	8245	455	7846	828	8245	331	7846	114
4	8144	364	8144	100	7059	136	8144	1700	7059	286	8145	1883	7059	360	7059	223
5	13117	1136	8334	262	8464	250	8334	496	8464	209	8334	514	8464	419	8334	77
6	11600	807	6415	275	6417	231	6415	946	6417	263	6416	755	6417	438	6415	107
7	13570	1019	10531	137	10531	216	10536	1521	10531	403	10536	682	10531	404	10531	90
8	6028	350	6028	61	6014	160	6028	393	6014	235	6028	510	6014	338	6014	213
9	11908	656	7585	247	7688	350	7585	659	7688	322	7585	936	7688	427	7585	72
10	13032	1412	9244	227	9285	247	9244	1430	9285	401	9245	375	9285	519	9244	108

 $\Sigma si=40.0$ 

1	7178	361	7178	75	6910	179	7178	422	6910	333	7178	579	6910	312	6910	73
2	11324	595	7320	72	7705	146	7320	344	7705	264	7320	363	7705	445	7320	105
3	12913	1001	9211	271	9445	359	9211	1115	9445	409	9211	564	9445	640	9211	114
4	5627	370	5627	80	4897	175	5627	617	4897	444	5627	659	4897	433	4897	216
5	13314	1202	10032	265	10038	196	10032	870	10038	555	10032	379	10038	389	10032	98
6	11579	713	7751	191	7751	153	7751	412	7751	242	7751	358	7751	191	7751	77
7	13349	784	8776	105	8992	212	8776	769	8992	415	8778	707	8992	400	8776	94
8	7047	360	7047	60	7030	127	7047	1746	7030	327	7048	531	7030	380	7030	55
9	7756	456	6395	89	6452	267	6395	1018	6452	348	6395	654	6452	279	6395	86
10	11095	1430	6240	176	6241	258	6240	523	6241	292	6241	432	6241	450	6240	85

 $\Sigma si=50.0$ 

1	9408	372	9408	115	9052	159	9408	1884	9052	315	9408	1699	9052	338	9052	109
2	11036	563	6856	63	7762	160	6856	200	7762	708	6856	1833	7762	446	6856	56
3	12735	826	9131	385	10291	237	9131	598	10291	512	9131	641	10291	462	9131	75
4	5819	334	5819	81	5060	302	5819	447	5060	581	5821	521	5060	405	5060	60
5	10988	1025	6345	181	6349	189	6345	562	6349	256	6345	816	6349	258	6345	81
6	13201	1608	7785	189	7786	267	7785	1085	7786	421	7788	1246	7786	398	7785	83
7	11201	1503	6909	145	7070	269	6909	353	7070	482	6909	285	7070	694	6909	64
8	9931	474	9143	76	9407	227	9143	584	9407	534	9143	639	9407	540	9143	72
9	5630	385	5627	80	4897	175	5627	617	4897	444	5627	659	4897	433	4897	216
10	10650	1556	5822	188	5945	264	5822	465	5945	248	5824	723	5945	427	5822	72

## STYLE 1, VARIATION 2(20X20X20, $\Sigma_{cap}=6.0$ )

S/NO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	9528	3326	6793	851	408	1519	6805	4464	408	1117	6817	10621	408	1022
2	11298	7990	5925	1663	404	1553	5939	8215	404	1099	5953	10389	404	1183
3	12594	5681	6757	1736	405	1021	6775	3223	405	823	6782	5653	405	1054
4	11202	9004	5645	1024	405	1251	5656	6542	405	1459	5666	8468	405	1322
5	8120	4615	6161	316	408	1467	6166	4302	408	1031	6173	9034	408	1524
6	11763	8120	6047	1572	408	1635	6062	5300	408	1259	6074	8741	408	1805
7	13740	13304	6265	1796	406	1876	6278	4463	406	1317	6292	10447	406	939
8	11865	9875	5824	2245	405	1203	5837	5114	405	1409	5850	9353	405	1696
9	10340	7244	5875	1335	407	1378	5886	5241	407	988	5896	8346	407	927
10	6495	1735	6402	438	409	1379	6407	4622	409	1035	6410	7820	409	2365
<b><math>\Sigma_{si}=1.5</math></b>														
1	8722	3923	5567	496	403	1696	5574	2827	403	759	5578	6639	403	1030
2	13273	8831	6540	1610	405	1083	6554	5155	405	794	6564	6118	405	1268
3	12277	8343	6053	997	403	1187	6071	3865	403	1181	6090	5920	403	1052
4	8861	3397	6749	1315	407	1948	6768	4481	407	1035	6779	6201	407	1138
5	12091	5507	6231	610	404	1102	6242	3475	404	1435	6250	4921	404	1566
6	12098	7532	5996	1432	403	1220	6005	3461	403	1002	6013	5711	403	1014
7	11389	5298	5769	239	403	1346	5770	2095	403	1106	5772	2222	403	1103
8	7596	3132	5966	836	406	1338	5976	2919	406	940	5984	6069	406	1423
9	11298	7990	5925	1663	404	1553	5939	8215	404	1099	5953	10389	404	1183
10	12082	7085	5669	1211	404	996	5684	5631	404	1055	5694	7095	404	1287
<b><math>\Sigma_{si}=2.0</math></b>														
1	7317	2285	6299	1759	405	1536	6315	2335	405	872	6324	4777	405	906
2	12594	5681	6757	1736	405	1021	6775	3223	405	823	6782	5653	405	1054
3	11878	4178	5961	448	404	1328	5965	3732	404	800	5967	5788	404	1107
4	8898	1631	5526	192	404	300	5530	1186	404	356	5530	1327	404	722
5	11727	3602	5882	251	404	1499	5888	4131	404	739	5889	4323	404	705
6	10884	4763	5573	569	406	1163	5582	3597	406	1396	5588	4995	406	1715
7	11668	7010	5833	645	404	650	5845	3772	404	863	5853	7059	404	1134
8	5524	1536	5518	346	404	897	5524	1909	404	900	5524	2121	404	740
9	7413	3601	6137	411	404	994	6141	3570	404	997	6144	6730	404	1441
10	11267	4023	5556	160	405	1146	5557	2191	405	928	5557	2125	405	893
<b><math>\Sigma_{si}=5.0</math></b>														
1	9804	2819	6148	271	405	637	6157	2851	405	506	6161	2851	405	604
2	13565	5415	6796	646	402	642	6811	3004	402	669	6815	3987	402	553
3	12952	5029	6410	469	404	949	6424	2474	404	691	6426	2952	404	1146
4	6293	1658	5982	264	403	857	5996	1716	403	551	6000	3092	403	748
5	11993	4308	6046	810	404	1167	6061	2683	404	741	6064	3133	404	728
6	12594	5681	6757	1736	405	1021	6775	3223	405	823	6782	5653	405	1054
7	12377	4298	6324	543	403	385	6330	2744	403	669	6331	2181	403	822
8	6358	923	6057	444	404	611	6066	2058	404	767	6067	2855	404	885
9	11668	7010	5833	645	404	650	5845	3772	404	863	5853	7059	404	1134
10	10938	3955	5634	915	403	732	5645	2192	403	512	5647	3255	403	861

$\Sigma si=10.0$ 

1	6084	1329	6056	105	402	663	6067	1676	402	509	6069	1135	402	578
2	13043	3040	6333	283	402	234	6348	1451	402	280	6350	2361	402	471
3	10796	2613	5595	178	405	1033	5601	2387	405	448	5602	2356	405	566
4	6959	1572	5587	245	404	491	5601	1703	404	375	5604	1798	404	652
5	11249	3320	5660	420	404	474	5673	1647	404	545	5674	3009	404	793
6	10836	3212	5686	255	403	779	5695	2792	403	500	5696	3077	403	736
7	6293	1658	5982	264	403	857	5996	1716	403	551	6000	3092	403	748
8	6211	1055	5609	241	403	433	5617	2046	403	368	5618	2210	403	735
9	6449	891	6276	116	403	420	6285	1890	403	490	6287	2507	403	530
10	10782	3064	5511	226	403	1482	5517	2133	403	459	5517	1686	403	912

 $\Sigma si=20.0$ 

1	7034	1209	5892	260	404	1031	5902	1222	404	385	5902	1038	404	461
2	11323	3406	5764	104	403	510	5769	1227	403	355	5769	940	403	644
3	11912	3341	5878	688	403	1245	5892	1425	403	415	5893	3411	403	556
4	6007	893	5848	80	403	322	5854	1105	403	444	5854	2016	403	756
5	12012	3096	6269	441	402	1319	6283	2271	402	505	6284	1275	402	701
6	12135	2272	6816	311	403	989	6820	1925	403	447	6821	2148	403	711
7	6960	1572	5587	245	404	491	5601	1703	404	375	5604	1798	404	652
8	8726	1435	5582	439	402	294	5590	1849	402	588	5590	2888	402	792
9	6564	1277	6500	233	404	572	6510	2448	404	419	6510	2023	404	527
10	11832	1552	5945	764	402	1229	5960	2265	402	525	5962	1818	402	877

 $\Sigma si=30.0$ 

1	9431	1271	6300	260	404	475	6313	1557	404	405	6314	1189	404	667
2	13354	4076	6516	423	404	436	6526	2172	404	409	6527	1553	404	765
3	11577	2435	5884	264	405	1159	5894	1587	405	514	5894	2564	405	432
4	8145	1421	6333	220	403	282	6348	1492	403	408	6349	2244	403	571
5	13117	3318	6190	532	403	873	6207	1817	403	363	6207	1971	403	488
6	11601	1961	6199	387	406	1030	6210	1359	406	496	6211	2317	406	537
7	13571	3888	6417	521	403	475	6430	1610	403	485	6431	1582	403	713
8	6028	958	5854	100	403	613	5863	1185	403	407	5863	2539	403	950
9	11910	3341	5878	688	403	1245	5892	1425	403	415	5893	3411	403	556
10	13033	3957	5768	866	403	549	5781	3448	403	573	5782	3110	403	642

 $\Sigma si=40.0$ 

1	7178	1378	5887	376	403	416	5900	1666	403	420	5901	2518	403	387
2	11324	2175	5885	204	402	602	5892	1730	402	625	5892	2090	402	668
3	12913	3679	5747	321	403	1095	5756	1522	403	553	5756	2191	403	520
4	5627	1068	5587	93	404	630	5595	1141	404	459	5595	2298	404	537
5	13414	4347	5948	404	403	547	5960	1359	403	540	5960	1856	403	798
6	11579	2437	6031	290	403	1072	6043	1475	403	307	6043	2984	403	591
7	13355	4076	6516	423	404	436	6526	2172	404	409	6527	1553	404	765
8	7048	1093	6416	114	402	544	6429	1554	402	346	6430	2319	402	504
9	7756	1319	5575	161	404	325	5580	1434	404	920	5580	1405	404	776
10	11096	2652	5900	370	403	531	5909	2146	403	492	5910	2460	403	648

 $\Sigma si=50.0$ 

1	9408	1595	6891	195	404	391	6907	1593	404	354	6909	1426	404	415
2	11036	2319	5579	168	404	550	5585	1496	404	428	5585	2541	404	442
3	12735	3622	5691	232	404	1225	5703	1748	404	377	5703	2621	404	718
4	5821	897	5799	103	405	304	5808	1217	405	383	5808	2730	405	516
5	10998	2361	5642	155	404	708	5647	1786	404	369	5648	1740	404	537
6	13204	3027	6723	370	403	626	6734	1708	403	584	6735	1075	403	580
7	11201	2702	5799	118	403	568	5805	2346	403	409	5805	2411	403	568
8	9931	1737	6000	174	404	461	6008	1635	404	446	6008	1202	404	466
9	5625	1068	5587	93	404	630	5595	1141	404	459	5595	2298	404	537
10	10561	2475	5540	228	403	273	5550	3073	403	339	5551	3155	403	921

## STYLE 2, VARIATION 2(20X20X20, $\Sigma_{cap}=6.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER
<b><math>\Sigma_{si}=1.0</math></b>														
1	9504	944	6777	235	404	144	6790	468	404	210	6817	1336	404	257
2	11287	2006	5921	205	403	113	5935	445	403	250	5953	2839	403	366
3	12590	2787	6754	222	404	138	6772	456	404	521	6782	2221	404	400
4	11187	3317	5635	191	403	146	5645	511	403	288	5666	1126	403	442
5	8096	1156	6146	215	404	105	6151	538	404	359	6173	1043	404	280
6	11748	2807	6038	197	405	145	6052	616	405	553	6074	1349	405	303
7	13725	4897	6257	272	402	105	6271	549	402	221	6292	1228	402	200
8	11849	3306	5816	339	403	143	5828	478	403	319	5850	1153	403	606
9	10136	1556	5860	293	403	114	5871	527	403	198	5896	3185	403	297
10	6461	381	6374	193	405	153	6378	556	405	313	6410	927	405	436
<b><math>\Sigma_{si}=1.5</math></b>														
1	8704	906	5559	164	402	126	5566	760	402	416	5578	862	402	293
2	13262	3898	6535	318	403	105	6549	495	403	288	6564	2226	403	216
3	12270	2882	6048	135	403	136	6065	482	403	158	6080	2298	403	197
4	8851	1202	6745	356	406	135	6764	526	406	406	6779	854	406	284
5	12081	2062	6223	147	403	119	6234	470	403	313	6250	2377	403	281
6	12090	3546	5988	213	402	129	5998	547	402	243	6013	2549	402	307
7	11378	2105	5757	113	403	128	5758	462	403	285	5772	2521	403	454
8	7582	797	5961	242	403	102	5971	556	403	138	5984	1213	403	222
9	11285	2106	5921	205	403	113	5935	445	403	250	5953	2839	403	366
10	12063	3091	5661	175	402	129	5676	454	402	375	5694	2881	402	266
<b><math>\Sigma_{si}=2.0</math></b>														
1	7307	699	6298	180	404	134	6315	520	404	381	6324	2367	404	321
2	12592	2787	6754	222	404	138	6772	456	404	521	6782	2221	404	400
3	11872	2409	5953	188	403	122	5967	677	403	313	5967	2076	403	255
4	8898	1228	5525	129	404	132	5530	450	404	375	5530	632	404	248
5	11724	1893	5878	195	403	146	5883	474	403	188	5889	798	403	301
6	10879	2093	5567	147	405	127	5576	542	405	456	5588	1055	405	271
7	11665	3449	5831	354	403	124	5842	466	403	485	5853	1198	403	420
8	5519	298	5514	89	402	112	5519	378	402	170	5524	645	402	365
9	7400	1021	6135	195	403	157	6139	540	403	478	6144	2368	403	374
10	11252	1657	5541	270	404	137	5542	529	404	234	5557	1745	404	281
<b><math>\Sigma_{si}=5.0</math></b>														
1	9793	1804	6143	224	404	127	6152	552	404	415	6161	666	404	275
2	13563	4675	6795	128	402	123	6811	493	402	274	6815	615	402	226
3	12951	3718	6410	218	403	130	6423	497	403	394	6426	1595	403	282
4	6291	582	5980	261	402	111	5994	483	402	332	6000	705	402	293
5	11985	3523	6046	247	404	115	6061	569	404	501	6064	1860	404	253
6	12590	2787	6754	222	404	138	6772	456	404	521	6782	2221	404	400
7	12373	2988	6321	211	403	152	6327	476	403	288	6331	1024	403	420
8	7017	712	5843	216	401	117	5855	487	401	116	5860	1888	401	435
9	11658	3449	5831	354	403	124	5842	466	403	485	5853	1198	403	420
10	10936	2829	5632	213	403	119	5642	705	403	249	5647	888	403	302

$\Sigma si=10.0$ 

1	6078	382	6056	121	402	113	6067	459	402	471	6069	640	402	285
2	13042	3038	6333	220	402	149	6348	522	402	127	6350	765	402	197
3	10796	2018	5595	131	405	130	5601	494	405	458	5602	1591	405	184
4	6958	732	5587	137	404	120	5601	452	404	202	5604	744	404	270
5	11249	2560	5660	195	404	134	5673	452	404	209	5674	1361	404	175
6	10830	2922	5682	264	403	122	5691	622	402	268	5696	835	403	265
7	6290	586	5980	261	402	111	5994	483	402	332	6000	705	402	293
8	6211	375	5609	132	403	113	5616	485	403	159	5618	1420	403	239
9	6448	519	6275	186	403	148	6285	437	403	613	6287	1340	403	429
10	10781	2510	5510	128	402	108	5515	465	402	293	5517	893	402	259

 $\Sigma si=20.0$ 

1	7034	510	5892	189	404	126	5902	528	404	261	5902	537	404	335
2	11323	2802	5764	82	403	127	5769	454	403	246	5769	1283	403	248
3	11912	3560	5878	194	403	110	5892	610	403	284	5893	775	403	291
4	6007	376	5848	132	403	173	5854	428	403	487	5854	564	403	358
5	12011	2760	6269	212	402	117	6283	476	402	224	6284	546	402	275
6	12135	1640	6816	162	403	139	6820	521	403	346	6821	828	403	286
7	6956	736	5587	137	404	120	5601	452	404	202	5604	744	404	270
8	8726	1073	5582	229	402	102	5590	628	402	142	5590	541	402	250
9	6564	382	6500	246	404	134	6510	501	404	297	6510	635	404	420
10	11832	3391	5945	277	402	110	5960	481	402	341	5962	819	402	274

 $\Sigma si=30.0$ 

1	9431	865	6299	213	404	142	6312	459	404	522	6314	737	404	250
2	13353	4365	6516	215	404	131	6526	496	404	201	6527	661	404	282
3	11577	2075	5884	203	405	152	5894	678	405	461	5894	729	405	324
4	8144	1328	6333	289	403	119	6348	540	403	483	6349	773	403	309
5	13117	4210	6190	235	403	119	6207	534	403	325	6207	632	403	306
6	11600	1761	6199	210	406	127	6210	458	406	193	6211	673	406	321
7	13570	3663	6417	220	403	143	6430	460	403	684	6431	666	403	323
8	6028	360	5845	198	403	111	5863	677	403	239	5863	629	403	187
9	11908	3566	5878	194	403	110	5892	610	403	284	5893	775	403	291
10	13032	4163	5768	244	402	152	5781	711	402	418	5782	870	402	289

 $\Sigma si=40.0$ 

1	7178	822	5887	211	403	135	5900	462	403	242	5901	726	403	250
2	11324	1900	5885	116	402	109	5892	589	402	175	5892	699	402	242
3	12913	5081	5746	212	403	115	5756	604	403	175	5756	802	403	365
4	5627	451	5587	168	404	166	5595	486	404	468	5595	549	404	393
5	13314	4528	5947	239	403	105	5960	511	403	274	5960	695	403	308
6	11579	3022	6031	237	403	143	6043	587	403	189	6043	795	403	238
7	13349	4344	6516	215	404	131	6526	496	404	201	6527	661	404	282
8	7047	540	6416	165	402	110	6429	563	402	355	6430	620	402	251
9	7756	957	5575	226	404	145	5580	634	404	394	5580	760	404	426
10	11095	2256	5900	169	403	134	5909	641	403	156	5910	820	403	357

 $\Sigma si=50.0$ 

1	9408	890	6891	298	404	121	6907	532	404	255	6909	792	404	350
2	11036	1829	5579	116	404	96	5585	439	404	168	5585	558	404	259
3	12735	4544	5691	270	404	163	5703	511	404	431	5703	836	404	269
4	5819	506	5799	121	405	101	5808	436	405	355	5808	644	405	236
5	10988	2535	5642	201	404	160	5647	555	404	199	5648	1113	404	226
6	13201	3171	6722	181	403	143	6734	581	403	534	6735	700	403	425
7	11201	2591	5799	173	403	126	5805	667	403	550	5805	804	403	402
8	9931	1317	6000	215	403	97	6008	508	403	246	6008	539	403	248
9	5630	481	5587	168	404	166	5595	486	404	468	5595	549	404	393
10	10650	2035	5540	213	403	96	5550	740	403	193	5551	1597	403	193

## **APPENDIX 2**



## STYLE 1, VARIATION 2(20X20X20, $\Sigma_{cap}=2.0$ )

SNO	FORMULATION 1A		FORMULATION 1B		FORMULATION 2B		FORMULATION 3B		FORMULATION 4B		FORMULATION 5B		FORMULATION 6B		NEW FORMULATION	
	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER	OBJ	ITER

### $\Sigma_{si}=1.0$

1	39254	27637	7965	1613	411	1815	7984	5189	411	983	7999	8269	411	1407	7984	2427
2	40624	141003	6774	2874	405	2055	6791	8251	405	1145	6808	9239	405	1257	6791	3158
3	38280	30028	7118	2616	406	1782	7137	6367	406	1039	7147	6966	406	1181	7137	2125
4	36590	41022	7035	2139	409	2458	7055	7261	409	1335	7072	9527	409	1544	7055	3170
5	31584	37775	6086	2172	407	2218	6101	4366	407	1224	6115	7466	407	2168	6101	6689
6	33479	28449	6593	2219	410	2217	6607	5233	410	1136	6622	9360	410	1382	6607	3629
7	32174	31092	6704	1950	408	1771	6719	6422	408	1610	6733	10489	408	2093	6719	7049
8	30618	25153	6429	2038	407	1552	6445	5221	407	1588	6461	7944	407	1533	6445	3518
9	27710	16325	6270	2798	406	2309	6286	3890	406	2275	6302	9038	406	1760	6286	3516
10	27688	13619	6510	3135	411	1701	6523	8231	411	1873	6540	8222	411	1183	6523	3543

### $\Sigma_{si}=5.0$

1	32269	12025	6874	777	406	957	6887	3304	406	639	6889	4118	406	677	6887	1948
2	36121	15297	7257	1326	403	1060	7274	3378	403	959	7279	4654	403	676	7274	2972
3	35617	14010	7156	1698	405	1426	7180	4971	405	1034	7172	2965	405	1306	7169	1460
4	33509	12564	6510	1072	404	1696	6528	2304	404	1003	6531	3952	404	876	6528	1526
5	34714	13654	6789	1667	405	933	6804	3387	405	766	6807	3446	405	710	6804	1592
6	38284	30028	7118	2616	406	1782	7137	6367	406	1039	7147	6966	406	1181	7137	5230
7	36967	16807	7032	2256	405	1454	7047	2514	405	554	7051	3431	405	768	7047	1679
8	34917	14180	7037	1484	404	1193	7050	3152	404	718	7052	3136	404	999	7050	1733
9	28146	5291	6961	874	407	951	6975	2622	407	1044	6977	2895	407	975	6975	2662
10	38890	42899	6437	473	404	1082	6455	1808	404	616	6457	3717	404	974	6455	2469

### $\Sigma_{si}=10.0$

1	30004	7729	6872	860	403	1067	6889	1642	403	696	6892	2632	403	1148	6889	3104
2	45447	41199	7312	588	403	585	7330	1704	403	497	7332	2002	403	833	7330	2448
3	32874	12316	6511	1361	408	1382	6525	1923	408	575	6526	3104	408	778	6525	1586
4	25009	5603	6206	625	406	849	6221	2133	406	463	6222	2274	406	974	6221	1427
5	30654	6347	7061	724	405	593	7077	1988	405	766	7078	3359	405	1041	7077	1244
6	29343	7971	6175	995	403	1191	6190	1911	403	741	6192	3126	403	686	6190	1922
7	33510	12564	6510	1072	404	1696	6528	2304	404	1003	6531	3952	404	876	6528	1526
8	34897	14563	6260	659	405	1052	6276	3055	405	626	6277	1772	405	639	6276	1226
9	28986	5902	6735	965	404	1606	6753	2202	404	564	6754	2844	404	856	6753	2489
10	29286	10250	6050	1281	403	1069	6066	2199	403	491	6067	1741	403	654	6066	3058

### $\Sigma_{si}=20.0$

1	26468	3899	6759	825	405	1695	6777	1705	405	490	6777	2542	405	1004	6777	1736
2	24250	3708	6083	543	404	852	6097	1923	404	550	6097	2450	404	762	6097	1074
3	35130	16114	6611	596	404	1621	6626	2082	404	594	6627	1573	404	888	6626	1489
4	31387	10874	6252	1661	404	1715	6270	1420	404	555	6270	2872	404	947	6270	1292
5	32971	7453	6695	699	403	1801	6709	2503	403	730	6709	2827	403	1308	6709	1943
6	27546	3513	6889	573	405	993	6905	1244	405	630	6905	1675	405	694	6905	1271
7	25010	5603	6206	625	406	849	6221	2133	406	463	6222	2274	406	974	6221	1427
8	38457	14628	7095	514	405	452	7097	1429	405	685	7111	3134	405	579	7111	2665
9	34523	10572	6854	791	405	613	6872	1784	405	447	6872	2835	405	729	6872	2510
10	28884	8571	6605	836	405	1118	6622	2067	405	939	6622	1288	405	867	6622	1317



$\Sigma si=50.0$

1	32816	11232	7773	794	404	835	7789	4127	404	500	7789	1359	404	853	7789	1257
2	36136	16523	6747	932	406	1353	6764	1539	405	503	6764	2676	406	556	6764	2394
3	34881	12548	7324	766	405	1103	7343	2154	405	504	7343	2013	405	559	7343	1259
4	31262	7824	6006	715	405	531	6020	1461	405	533	6020	1865	405	632	6020	2658
5	25294	4081	5929	812	406	1009	5944	1947	406	431	5944	2540	406	661	5944	2541
6	30099	5278	7097	542	404	1118	7109	1472	404	514	7109	1213	404	1056	7109	1275
7	32782	12564	6573	457	404	886	6589	1252	404	540	6589	2550	404	731	6589	2526
8	35721	21598	6805	1107	405	1154	6824	2194	405	676	6824	1257	405	507	6824	1400
9	18636	2386	5578	215	406	1414	5588	1438	405	732	5589	1561	406	789	5588	1249
10	29747	68407	6152	906	405	682	6167	2117	405	463	6168	1239	405	916	6167	1400

## **APPENDIX 3**

## STYLE 1 VARIATION 1

### FORMULATION S1V1.1(A)

SETS:  
SUPPLY/1..20/:S;  
WAREHOUSE/1..20/: CAP, F,Y;  
DEMAND/1..20/:D;  
TRANSPORT (SUPPLY,DEMAND,WAREHOUSE) :C,X;  
DUMMY1 (SUPPLY,DEMAND) ;  
DUMMY2 (WAREHOUSE,DEMAND) ;  
DUMMY3 (SUPPLY,WAREHOUSE) ;  
ENDSETS

DATA:

ENDDATA

MIN=@SUM (TRANSPORT (I,J,K) :C (I,J,K) \*X (I,J,K) ) +  
@SUM (WAREHOUSE (J) :F (J) \*Y (J) ) ;

!SUBJECT TO

@SUM (TRANSPORT (I,J,K) :X (I,J,K) )=1;  
@FOR (SUPPLY (I) :@SUM (DUMMY2 (J,K) :X (I,J,K) ) <=S (I) ) ;  
@FOR (DEMAND (K) :@SUM (DUMMY3 (I,J) :X (I,J,K) ) =D (K) ) ;  
@FOR (TRANSPORT (I,J,K) :X (I,J,K) >=0) ;  
@FOR (WAREHOUSE (J) :@SUM (DUMMY3 (I,K) :X (I,J,K) ) <=CAP (J) \*Y (J) ) ;  
@FOR (WAREHOUSE (J) :@SUM (DUMMY3 (I,K) :X (I,J,K) ) <=Y (J) ) ;  
@FOR (WAREHOUSE (J) :@BIN (Y (J) ) ) ;

### FORMULATION S1V1.1(B)

SETS:  
SUPPLY/1..20/:S;  
WAREHOUSE/1..20/:CAP,F,Y;  
DEMAND/1..20/:D;  
TRANSPORT (SUPPLY,DEMAND,WAREHOUSE) :C,X;  
DUMMY1 (SUPPLY,DEMAND) ;  
DUMMY2 (WAREHOUSE,DEMAND) ;  
DUMMY3 (SUPPLY,WAREHOUSE) ;  
ENDSETS

DATA:

ENDDATA

MIN=@SUM (TRANSPORT (I,J,K) :C (I,J,K) \*X (I,J,K) ) +  
@SUM (WAREHOUSE (J) :F (J) \*Y (J) ) ;

!SUBJECT TO

```
@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) = 1;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) );
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) );
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0);
@FOR (WAREHOUSE (J) : @SUM (DUMMY3 (I, K) : X (I, J, K) ) <= CAP (J) * Y (J) );
@FOR (WAREHOUSE (J) : @SUM (DUMMY3 (I, K) : X (I, J, K) ) <= Y (J) );
@FOR (WAREHOUSE (J) : Y (J) >= 0);
```

#### FORMULATION S1V1.2 (B)

SETS:

```
SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP, F, Y;
DEMAND/1..20/:D;
TRANSPORT (SUPPLY, DEMAND, WAREHOUSE) : C, X;
DUMMY1 (SUPPLY, DEMAND);
DUMMY2 (WAREHOUSE, DEMAND);
DUMMY3 (SUPPLY, WAREHOUSE);
ENDSETS
```

DATA:

M=100000;

ENDDATA

```
MIN=@SUM (TRANSPORT (I, J, K) : C (I, J, K) * X (I, J, K) ) +
      @SUM (WAREHOUSE (J) : F (J) * Y (J) );
```

!SUBJECT TO

```
@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) = 1;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) );
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) );
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0);
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) <= CAP (J) * Y (J) );

@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) >= M * Y (J) - M);
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) >= -M * Y (J) );
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) <= M * Y (J) );
@FOR (WAREHOUSE (J) : Y (J) >= 0);
```

#### FORMULATION S1V1.3 (B)

SETS:

```
SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP, F, Y;
DEMAND/1..20/:D;
TRANSPORT (SUPPLY, DEMAND, WAREHOUSE) : C, X;
DUMMY1 (SUPPLY, DEMAND);
DUMMY2 (WAREHOUSE, DEMAND);
```

```
DUMMY3 (SUPPLY, WAREHOUSE) ;
ENDSETS
```

```
DATA:
```

```
ENDDATA
```

```
MIN=@SUM (TRANSPORT (I, J, K) : C (I, J, K) *X (I, J, K) ) +
      @SUM (WAREHOUSE (J) : F (J) *Y (J) ) ;
```

```
!SUBJECT TO
```

```
@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) =1 ;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) ) ;
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) ) ;
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0) ;
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) <= CAP (J) *Y (J) ) ;
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) <= D (K) *Y (J) ) ) ;

@FOR (WAREHOUSE (J) : Y (J) >= 0) ;
```

#### FORMULATION S1V1.4 (B)

```
SETS:
```

```
SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP,F,Y;
DEMAND/1..20/:D;
TRANSPORT (SUPPLY, DEMAND, WAREHOUSE) : C,X;
DUMMY1 (SUPPLY, DEMAND) ;
DUMMY2 (WAREHOUSE, DEMAND) ;
DUMMY3 (SUPPLY, WAREHOUSE) ;
ENDSETS
```

```
DATA:
```

```
M=100000;
ENDDATA
```

```
MIN=@SUM (TRANSPORT (I, J, K) : C (I, J, K) *X (I, J, K) ) +
      @SUM (WAREHOUSE (J) : F (J) *Y (J) ) ;
```

```
!SUBJECT TO
```

```
@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) =1 ;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) ) ;
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) ) ;
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0) ;
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) <= CAP (J) *Y (J) ) ;

@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) <= -
M*Y (J) + M + D (K) ) ) ;
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) >= -M*Y (J) ) ) ;
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) <= M*Y (J) ) ) ;
@FOR (WAREHOUSE (J) : Y (J) >= 0) ;
```

### FORMULATION S1V1.5(B)

SETS:

SUPPLY/1..20/:S;

WAREHOUSE/1..20/:CAP,F,Y;

DEMAND/1..20/:D;

TRANSPORT(SUPPLY,DEMAND,WAREHOUSE):C,X;

DUMMY1(SUPPLY,DEMAND);

DUMMY2(WAREHOUSE,DEMAND);

DUMMY3(SUPPLY,WAREHOUSE);

ENDSETS

DATA:

ENDDATA

MIN=@SUM(TRANSPORT(I,J,K):C(I,J,K)\*X(I,J,K))+  
@SUM(WAREHOUSE(J):F(J)\*Y(J));

!SUBJECT TO

@SUM(TRANSPORT(I,J,K):X(I,J,K))=1;

@FOR(SUPPLY(I):@SUM(DUMMY2(J,K):X(I,J,K))<=S(I));

@FOR(DEMAND(K):@SUM(DUMMY3(I,J):X(I,J,K))=D(K));

@FOR(TRANSPORT(I,J,K):X(I,J,K)>=0);

@FOR(WAREHOUSE(J):@SUM(DUMMY1(I,K):X(I,J,K))<=CAP(J)\*Y(J));

@FOR(DEMAND(K):@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X(I,J,K))<=D(K)\*Y(J)));

@FOR(WAREHOUSE(J):@FOR(SUPPLY(I):@SUM(DEMAND(K):X(I,J,K))<=S(I)\*Y(J)));

@FOR(WAREHOUSE(J):Y(J)>=0);

### FORMULATION S1V1.6(B)

SETS:

SUPPLY/1..20/:S;

WAREHOUSE/1..20/:CAP,F,Y;

DEMAND/1..20/:D;

TRANSPORT(SUPPLY,DEMAND,WAREHOUSE):C,X;

DUMMY1(SUPPLY,DEMAND);

DUMMY2(WAREHOUSE,DEMAND);

DUMMY3(SUPPLY,WAREHOUSE);

ENDSETS

DATA:

M=100000;

ENDDATA

MIN=@SUM(TRANSPORT(I,J,K):C(I,J,K)\*X(I,J,K))+  
@SUM(WAREHOUSE(J):F(J)\*Y(J));

!SUBJECT TO

```

@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) = 1;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) );
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) );
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0 );
@FOR (WAREHOUSE (J) : @SUM (DUMMY1 (I, K) : X (I, J, K) ) <= CAP (J) * Y (J) );

@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) <= -
M * Y (J) + M + D (K) ) );
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) >= - M * Y (J) ) );
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X (I, J, K) ) <= M * Y (J) ) );

@FOR (WAREHOUSE (J) : @FOR (SUPPLY (I) : @SUM (DEMAND (K) : X (I, J, K) ) <= -
M * Y (J) + M + S (I) ) );
@FOR (WAREHOUSE (J) : @FOR (SUPPLY (I) : @SUM (DEMAND (K) : X (I, J, K) ) >= - M * Y (J) ) );
@FOR (WAREHOUSE (J) : @FOR (SUPPLY (I) : @SUM (DEMAND (K) : X (I, J, K) ) <= M * Y (J) ) );

@FOR (WAREHOUSE (J) : Y (J) >= 0 );

```

#### FORMULATION SIV1.7 (B)

```

SETS:
SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP,F,Y;
DEMAND/1..20/:D;
TRANSPORT (SUPPLY, DEMAND, WAREHOUSE) : C, X;
DUMMY1 (SUPPLY, DEMAND);
DUMMY2 (WAREHOUSE, DEMAND);
DUMMY3 (SUPPLY, WAREHOUSE);
ENDSETS

```

DATA:

ENDDATA

```

MIN=@SUM (TRANSPORT (I, J, K) : C (I, J, K) * X (I, J, K) ) +
@SUM (WAREHOUSE (J) : F (J) * Y (J) );

```

!SUBJECT TO

```

@SUM (TRANSPORT (I, J, K) : X (I, J, K) ) = 1;
@FOR (SUPPLY (I) : @SUM (DUMMY2 (J, K) : X (I, J, K) ) <= S (I) );
@FOR (DEMAND (K) : @SUM (DUMMY3 (I, J) : X (I, J, K) ) = D (K) );
@FOR (TRANSPORT (I, J, K) : X (I, J, K) >= 0 );
@FOR (WAREHOUSE (J) : @SUM (DUMMY3 (I, K) : X (I, J, K) ) <= CAP (J) * Y (J) );
@FOR (WAREHOUSE (J) : Y (J) >= 0 );

```

## STYLE 1 VARIATION 2

Style 1 Variation 2 has formulations same as the formulations for Style 1 Variation 1 except that the constraint (in LINGO)

```
@FOR(WAREHOUSE(J):@SUM(DUMMY3(I,K):X(I,J,K))<=CAP(J)*Y(J));
```

Is replaced by constraint

```
@FOR(WAREHOUSE(J):@SUM(DUMMY3(I,K):X(I,J,K))<=CAP(J));
```

Rest of the structure is identically same.



## STYLE 2 VARIATION 1

### FORMULATION S2V1.1(A)

SETS:

SUPPLY/1..20/:S;

WAREHOUSE/1..20/:CAP,F,Y;

DEMAND/1..20/:D;

SUPP\_WAR(SUPPLY,WAREHOUSE):C1,X1;

WAR\_DEM(WAREHOUSE,DEMAND):C2,X2;

ENDSETS

DATA:

ENDDATA

MIN=@SUM(SUPP\_WAR(I,J):C1(I,J)\*X1(I,J))+  
@SUM(WAR\_DEM(J,K):C2(J,K)\*X2(J,K))+  
@SUM(WAREHOUSE(J):F(J)\*Y(J));

!SUBJECT TO

@FOR(SUPPLY(I):@SUM(WAREHOUSE(J):X1(I,J))<=S(I));  
@FOR(DEMAND(K):@SUM(WAREHOUSE(J):X2(J,K))>=D(K));  
@SUM(WAREHOUSE(I,J):X1(I,J))=1;  
@SUM(WAR\_DEM(J,K):X2(J,K))=1;  
@FOR(WAREHOUSE(J):@FOR(SUPPLY(I):X1(I,J)>=0));  
@FOR(DEMAND(K):@FOR(WAREHOUSE(J):X2(J,K)>=0));  
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))=@SUM(DEMAND(K):X2(J,K)));  
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=CAP(J)\*Y(J));  
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=Y(J));  
@FOR(WAREHOUSE(J):@BIN(Y(J)));

### FORMULATION S2V1.1(B)

SETS:

SUPPLY/1..20/:S;

WAREHOUSE/1..20/:CAP,F,Y;

DEMAND/1..20/:D;

SUPP\_WAR(SUPPLY,WAREHOUSE):C1,X1;

WAR\_DEM(WAREHOUSE,DEMAND):C2,X2;

ENDSETS

DATA:

ENDDATA

MIN=@SUM(SUPP\_WAR(I,J):C1(I,J)\*X1(I,J))+  
@SUM(WAR\_DEM(J,K):C2(J,K)\*X2(J,K))+  
@SUM(WAREHOUSE(J):F(J)\*Y(J));

!SUBJECT TO

```

@FOR (SUPPLY (I) : @SUM (WAREHOUSE (J) : X1 (I, J) ) <= S (I) ) ;
@FOR (DEMAND (K) : @SUM (WAREHOUSE (J) : X2 (J, K) ) = D (K) ) ;
@SUM (SUPP_WAR (I, J) : X1 (I, J) ) = 1 ;
@SUM (WAR_DEM (J, K) : X2 (J, K) ) = 1 ;
@FOR (WAREHOUSE (J) : @FOR (SUPPLY (I) : X1 (I, J) >= 0) ) ;
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : X2 (J, K) >= 0) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) = @SUM (DEMAND (K) : X2 (J, K) ) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) <= CAP (J) * Y (J) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) <= Y (J) ) ;
@FOR (WAREHOUSE (J) : Y (J) >= 0) ;

```

#### FORMULATION S2V1.2 (B)

SETS:

```

SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP,F,Y;
DEMAND/1..20/:D;
SUPP_WAR (SUPPLY, WAREHOUSE) : C1, X1;
WAR_DEM (WAREHOUSE, DEMAND) : C2, X2;

```

ENDSETS

DATA:

M=100000;

ENDDATA

```

MIN=@SUM (SUPP_WAR (I, J) : C1 (I, J) * X1 (I, J) ) +
      @SUM (WAR_DEM (J, K) : C2 (J, K) * X2 (J, K) ) +
      @SUM (WAREHOUSE (J) : F (J) * Y (J) ) ;

```

!SUBJECT TO

```

@FOR (SUPPLY (I) : @SUM (WAREHOUSE (J) : X1 (I, J) ) <= S (I) ) ;
@FOR (DEMAND (K) : @SUM (WAREHOUSE (J) : X2 (J, K) ) = D (K) ) ;
@SUM (SUPP_WAR (I, J) : X1 (I, J) ) = 1 ;
@SUM (WAR_DEM (J, K) : X2 (J, K) ) = 1 ;
@FOR (WAREHOUSE (J) : @FOR (SUPPLY (I) : X1 (I, J) >= 0) ) ;
@FOR (DEMAND (K) : @FOR (WAREHOUSE (J) : X2 (J, K) >= 0) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) = @SUM (DEMAND (K) : X2 (J, K) ) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) <= CAP (J) * Y (J) ) ;

@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) >= M * Y (J) - M ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) >= -M * Y (J) ) ;
@FOR (WAREHOUSE (J) : @SUM (SUPPLY (I) : X1 (I, J) ) <= M * Y (J) ) ;
@FOR (WAREHOUSE (J) : Y (J) >= 0) ;

```

#### FORMULATION S2V1.3 (B)

SETS:

```

SUPPLY/1..20/:S;

```

```

WAREHOUSE/1..20/:CAP,F,Y;
DEMAND/1..20/:D;
SUPP_WAR(SUPPLY,WAREHOUSE):C1,X1;
WAR_DEM(WAREHOUSE,DEMAND):C2,X2;

```

ENDSETS

DATA:

ENDDATA

```

MIN=@SUM(SUPP_WAR(I,J):C1(I,J)*X1(I,J))+
      @SUM(WAR_DEM(J,K):C2(J,K)*X2(J,K))+
      @SUM(WAREHOUSE(J):F(J)*Y(J));

```

!SUBJECT TO

```

@FOR(SUPPLY(I):@SUM(WAREHOUSE(J):X1(I,J))<=S(I));
@FOR(DEMAND(K):@SUM(WAREHOUSE(J):X2(J,K))=D(K));
@SUM(SUPP_WAR(I,J):X1(I,J))=1;
@SUM(WAR_DEM(J,K):X2(J,K))=1;
@FOR(WAREHOUSE(J):@FOR(SUPPLY(I):X1(I,J)>=0));
@FOR(DEMAND(K):@FOR(WAREHOUSE(J):X2(J,K)>=0));
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))=@SUM(DEMAND(K):X2(J,K)));
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=CAP(J)*Y(J));
@FOR(WAR_DEM(J,K):X2(J,K)<=Y(J)*D(K));
@FOR(WAREHOUSE(J):Y(J)>=0);

```

#### FORMULATION S2V1.4(B)

SETS:

```

SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP,F,Y;
DEMAND/1..20/:D;
SUPP_WAR(SUPPLY,WAREHOUSE):C1,X1;
WAR_DEM(WAREHOUSE,DEMAND):C2,X2;

```

ENDSETS

DATA:

M=100000;

ENDDATA

```

MIN=@SUM(SUPP_WAR(I,J):C1(I,J)*X1(I,J))+
      @SUM(WAR_DEM(J,K):C2(J,K)*X2(J,K))+
      @SUM(WAREHOUSE(J):F(J)*Y(J));

```

!SUBJECT TO

```

@FOR(SUPPLY(I):@SUM(WAREHOUSE(J):X1(I,J))<=S(I));

```

```

@FOR (DEMAND (K) :@SUM (WAREHOUSE (J) :X2 (J, K) )=D (K) ) ;
@SUM (SUPP_WAR (I, J) :X1 (I, J) )=1;
@SUM (WAR_DEM (J, K) :X2 (J, K) )=1;
@FOR (WAREHOUSE (J) :@FOR (SUPPLY (I) :X1 (I, J) >=0) ) ;
@FOR (DEMAND (K) :@FOR (WAREHOUSE (J) :X2 (J, K) >=0) ) ;
@FOR (WAREHOUSE (J) :@SUM (SUPPLY (I) :X1 (I, J) )=@SUM (DEMAND (K) :X2 (J, K) ) ) ;
@FOR (WAREHOUSE (J) :@SUM (SUPPLY (I) :X1 (I, J) ) <=CAP (J) *Y (J) ) ;
@FOR (WAR_DEM (J, K) :X2 (J, K) <=-M*Y (J) +M+D (K) ) ;
@FOR (WAR_DEM (J, K) :X2 (J, K) <=M*Y (J) ) ;
@FOR (WAR_DEM (J, K) :X2 (J, K) >=-M*Y (J) ) ;
@FOR (WAREHOUSE (J) :Y (J) >=0) ;

```

#### FORMULATION5 S2V1. (B)

SETS:

```

SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP, F, Y;
DEMAND/1..20/:D;
SUPP_WAR (SUPPLY, WAREHOUSE) :C1, X1;
WAR_DEM (WAREHOUSE, DEMAND) :C2, X2;

```

ENDSETS

DATA:

ENDDATA

```

MIN=@SUM (SUPP_WAR (I, J) :C1 (I, J) *X1 (I, J) ) +
      @SUM (WAR_DEM (J, K) :C2 (J, K) *X2 (J, K) ) +
      @SUM (WAREHOUSE (J) :F (J) *Y (J) ) ;

```

!SUBJECT TO

```

@FOR (SUPPLY (I) :@SUM (WAREHOUSE (J) :X1 (I, J) ) <=S (I) ) ;
@FOR (DEMAND (K) :@SUM (WAREHOUSE (J) :X2 (J, K) )=D (K) ) ;
@SUM (SUPP_WAR (I, J) :X1 (I, J) )=1;
@SUM (WAR_DEM (J, K) :X2 (J, K) )=1;
@FOR (WAREHOUSE (J) :@FOR (SUPPLY (I) :X1 (I, J) >=0) ) ;
@FOR (DEMAND (K) :@FOR (WAREHOUSE (J) :X2 (J, K) >=0) ) ;
@FOR (WAREHOUSE (J) :@SUM (SUPPLY (I) :X1 (I, J) )=@SUM (DEMAND (K) :X2 (J, K) ) ) ;
@FOR (WAREHOUSE (J) :@SUM (SUPPLY (I) :X1 (I, J) ) <=CAP (J) *Y (J) ) ;
@FOR (WAR_DEM (J, K) :X2 (J, K) <=Y (J) *D (K) ) ;
@FOR (SUPP_WAR (I, J) :X1 (I, J) <=Y (J) *S (I) ) ;
@FOR (WAREHOUSE (J) :Y (J) >=0) ;

```

#### FORMULATION S2V1.6 (B)

SETS:

```

SUPPLY/1..20/:S;
WAREHOUSE/1..20/:CAP, F, Y;
DEMAND/1..20/:D;

```

SUPP\_WAR(SUPPLY, WAREHOUSE) : C1, X1;  
WAR\_DEM(WAREHOUSE, DEMAND) : C2, X2;

ENDSETS

DATA:

M=100000;

ENDDATA

MIN=@SUM(SUPP\_WAR(I, J) : C1(I, J) \* X1(I, J)) +  
@SUM(WAR\_DEM(J, K) : C2(J, K) \* X2(J, K)) +  
@SUM(WAREHOUSE(J) : F(J) \* Y(J));

!SUBJECT TO

@FOR(SUPPLY(I) : @SUM(WAREHOUSE(J) : X1(I, J)) <= S(I));  
@FOR(DEMAND(K) : @SUM(WAREHOUSE(J) : X2(J, K)) = D(K));  
@SUM(SUPP\_WAR(I, J) : X1(I, J)) = 1;  
@SUM(WAR\_DEM(J, K) : X2(J, K)) = 1;  
@FOR(WAREHOUSE(J) : @FOR(SUPPLY(I) : X1(I, J) >= 0));  
@FOR(DEMAND(K) : @FOR(WAREHOUSE(J) : X2(J, K) >= 0));  
@FOR(WAREHOUSE(J) : @SUM(SUPPLY(I) : X1(I, J)) = @SUM(DEMAND(K) : X2(J, K)));  
@FOR(WAREHOUSE(J) : @SUM(SUPPLY(I) : X1(I, J)) <= CAP(J) \* Y(J));

@FOR(WAR\_DEM(J, K) : X2(J, K) <= -M \* Y(J) + M + D(K));  
@FOR(WAR\_DEM(J, K) : X2(J, K) <= M \* Y(J));  
@FOR(WAR\_DEM(J, K) : X2(J, K) >= -M \* Y(J));

@FOR(SUPP\_WAR(I, J) : X1(I, J) <= -M \* Y(J) + M + S(I));  
@FOR(SUPP\_WAR(I, J) : X1(I, J) <= M \* Y(J));  
@FOR(SUPP\_WAR(I, J) : X2(I, J) >= -M \* Y(J));

@FOR(WAREHOUSE(J) : Y(J) >= 0);

#### FORMULATION S2V1.7(B)

SETS:

SUPPLY/1..20/:S;  
WAREHOUSE/1..20/:CAP, F, Y;  
DEMAND/1..20/:D;  
SUPP\_WAR(SUPPLY, WAREHOUSE) : C1, X1;  
WAR\_DEM(WAREHOUSE, DEMAND) : C2, X2;

ENDSETS

DATA:

ENDDATA

MIN=@SUM(SUPP\_WAR(I, J) : C1(I, J) \* X1(I, J)) +

```
@SUM(WAR_DEM(J,K):C2(J,K)*X2(J,K))+
@SUM(WAREHOUSE(J):F(J)*Y(J));
```

!SUBJECT TO

```
@FOR(SUPPLY(I):@SUM(WAREHOUSE(J):X1(I,J))<=S(I));
@FOR(DEMAND(K):@SUM(WAREHOUSE(J):X2(J,K))=D(K));
@SUM(SUPP_WAR(I,J):X1(I,J))=1;
@SUM(WAR_DEM(J,K):X2(J,K))=1;
@FOR(WAREHOUSE(J):@FOR(SUPPLY(I):X1(I,J)>=0));
@FOR(DEMAND(K):@FOR(WAREHOUSE(J):X2(J,K)>=0));
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))=@SUM(DEMAND(K):X2(J,K)));
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=CAP(J)*Y(J));
@FOR(WAREHOUSE(J):Y(J)>=0);
```

## STYLE 2 VARIATION 2

Style 1 Variation 2 has formulations same as the formulations for Style 1 Variation 1 except that the constraint (in LINGO)

```
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=CAP(J)*Y(J));
```

Is replaced by constraint

```
@FOR(WAREHOUSE(J):@SUM(SUPPLY(I):X1(I,J))<=CAP(J));
```

Rest of the structure is identically same.